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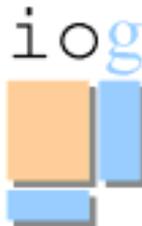
## STOCHASTIC ANALYSIS OF INPUT-OUTPUT MULTIPLIERS ON THE BASIS OF USE AND MAKE MATRICES.

**Rueda Cantuche, José Manuel**

Universidad Pablo de Olavide.

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In the literature, the construction of technical coefficients is linked to flow data (use and make matrices), but stochastics are imposed on the coefficients when multipliers are calculated, by means of the Leontief inverse. Due the nonlinearity of this operation, the multiplier estimates are biased (it is generally argued that the Leontief inverse underestimates input-output multipliers). By going back to the flow data, this paper provides unbiased and consistent employment and output multipliers estimates for the Andalusian economy. Rectangular use and make matrices are accommodated and technical coefficients, the Leontief inverse, and associated problems (such as negative coefficients) are circumvented.

**STOCHASTIC ANALYSIS OF INPUT-OUTPUT MULTIPLIERS ON THE  
BASIS OF USE AND MAKE MATRICES**

José Manuel Rueda-Cantuche

*Pablo de Olavide University at Seville, Spain*

[jmruecan@upo.es](mailto:jmruecan@upo.es)

Thijs ten Raa

*Tilburg University, the Netherlands*

**ABSTRACT:** In the literature, the construction of technical coefficients is linked to flow data (use and make matrices), but stochastics are imposed on the coefficients when multipliers are calculated, by means of the Leontief inverse. Due the nonlinearity of this operation, the multiplier estimates are biased (it is generally argued that the Leontief inverse underestimates input-output multipliers). By going back to the flow data, this paper provides unbiased and consistent employment and output multipliers estimates for the Andalusian economy. Rectangular use and make matrices are accommodated and technical coefficients, the Leontief inverse, and associated problems (such as negative coefficients) are circumvented.

**KEYWORDS:** Stochastic input-output analysis, input-output multipliers, use and make matrices.

**JEL CODES:** C67; D57; C13; E27

## 1. INTRODUCTION

Input-output analysis revolves around a matrix of technical coefficients  $A = (a_{ij})_{i,j = 1, \dots, n}$  (where  $n$  is the number of commodities). One literature links the  $A$ -matrix to flow data embedded in the national accounts (the use and make tables). Another literature uses  $A$ -matrices for economic analysis, particularly the determination of cumulated coefficients or multipliers by means of the Leontief inverse, including the study of the transmission of randomness. Unfortunately, the two bodies are quite disjunct. Stochastics imposed on the  $A$ -matrix in multiplier analysis is assumed, rather than derived from variations in the data on the basis of which it is constructed. Dietzenbacher (1995) relates information on flows to multiplier estimates, but the flows are inter-industry constructs rather than use and make data. Obviously, a reconciliation of the two bodies of literature is in order. We will analyze variations in the use and make tables and take them all the way to multiplier estimates.

A reason that this has not been done before may be that each of the previous steps has proven difficult. Both the construction of input-output coefficients and the derivation of multipliers constitute non-linear transformations. Not surprisingly, results are partial and problems persist, such as the problem of negatives in the construction of coefficients and the bias of the Leontief inverse in multiplier analysis. It seems a daunting task to compile these issues. To our pleasant surprise, however, the complications seem to neutralize each other. We will write out the reduced form of multipliers in terms of use and make tables (essentially solving out the input-output coefficients) and the issues dissolve. Two non-linearities kill each other and standard regression analysis finishes the job.

Both direct and cumulated input-output coefficients are published by statistical offices and often considered data. By extension, the estimation of multipliers is considered data construction, and the fulfillment of relationships suggested by the theoretical literature – such as invariance with respect to the units of measurement – an empirical issue. This point of view is not correct. Multipliers are theoretical constructs which can be used to answer “if then” questions. They are not directly observable. Basically, multipliers are cumulated

input-output coefficients and clarity about what these are is required. We assume that input-output coefficients measure the commodity requirements per unit of product, where the latter is a commodity as well. The best way to model such coefficients is according to the so-called commodity technology model. Other types of input-output coefficients exist and, at least in principle, would be amenable to our approach.

In next section, we discuss the transformation of flow data to technical coefficients (in terms of construction and of stochastics) and the use of the latter in economic impact analysis. The two steps are integrated in section 3, where variations in the use and make matrices are taken directly to confidence intervals for multipliers. Section 4 presents employment and output multipliers for the Andalusian economy and section 5 concludes.

## 2. FROM DATA TO COEFFICIENTS AND FROM COEFFICIENTS TO MULTIPLIERS

Modern input-output accounting makes a clear distinction between commodities  $i = 1, \dots, n$  and activities  $j = 1, \dots, m$ , although the origins can be traced back to Edmonston (1952). At the most disaggregated level, an activity represents a plant. Plant  $j$  uses inputs (both factor services and commodities) to make products (commodities). For reasons of national accounting, it is customary to list the inputs in the  $j$ -th column of *use* matrix  $U = (u_{ij})$  but the outputs in the  $j$ -th row of *make* matrix  $V = (v_{ji})$ . The requirements of input  $i$  by industry  $j$  are proportional to its products  $v_{jk}$ . If we assume that the proportions,  $a_{ik}$ , are independent of the industry, (the so-called commodity technology assumption), we obtain for the technical coefficients:

$$(1) \quad u_{ij} = \sum_{k=1}^n a_{ik} v_{jk} \quad \text{for all } i = 1, \dots, n \text{ and } j = 1, \dots, m.$$

An input-output matrix  $A$  has to fulfill this equation to achieve full consistency with fundamentals of input-output analysis according to Konijn and Steenge (1995) and Kop Jansen and ten Raa (1990) arrived at this conclusion on axiomatic grounds. If there are more activities than commodities ( $m > n$ ), the

system of equations (1) is overdetermined, an error term must be attached, and the input-output coefficients become regression coefficients.

Several studies have attempted to estimate technical coefficients from econometric models with cross-section data on firms' inputs and outputs. In some cases, these reduce to statistical tests for the reasonableness of the commodity technology assumption. For instance, Matthey and ten Raa (1997) proved for United States manufacturing that differences in material input factor intensities tend to reflect patterns of product specialization rather than coexisting technologies. The results support the assumption that material requirements for a product remain the same when it is produced as a secondary output. Econometric models have also been used with the purpose to show how the level of uncertainty in measuring the technical coefficients may be quantified. Using input-output data from the West Virginia economy, Miernyk (1970) shows that ordinary least squares and two-stages least squares estimates may be preferred to Durbin and Wald-Bartlett methods.

In the literature on stochastic input-output analysis, technical coefficients are the point of departure for the analysis of the probabilistic properties of the Leontief inverse,  $(I - A)^{-1}$ . The Leontief inverse is used to model the multiplier effects of a final demand stimulus on outputs, as well as to analyze multiplier effects of factor cost increases on prices. Kop Jansen (1994) reviews how stochastics affect the multipliers, i.e. the distributional properties of the Leontief inverse. The classical result is Simonovits's (1975) proof that if all the elements of  $A$  are independent, random and symmetrically distributed, then the expected value of the Leontief inverse is underestimated by the Leontief inverse of the expected value of  $A$ :

$$(2) \quad E[(I - A)^{-1}] \geq (I - E[A])^{-1}.$$

### **3. A DIRECT RELATIONSHIP BETWEEN MULTIPLIERS AND FLOW DATA**

An *output* multiplier is given by the total value of production needed to satisfy a euro worth of a particular component of final demand and *employment* multipliers

measure the associated number of workers. In what follows, we will assume the commodity technology hypothesis for intermediate and labor inputs. Use and make transactions are domestic and valued at basic prices. Net commodity taxes and not deductible Value Added Tax (VAT) are excluded, as are trade and transport margins. The latter are assigned to the trade and transport services industry. The measurement in basic prices accommodates the treatment of net exports as part of final demand. Since we will not use the interindustry format, we do not face the transition problem addressed by Jackson (1998).

### *Employment multipliers*

Commodity technology labor coefficients are determined by the following expression:

$$(3) \quad L = lV^T,$$

where  $L$  represents a row vector of labor employment (of order  $m$ ),  $l$  is the row vector of labor coefficients and  $V^T$  the transposed make matrix. Inflation by the Leontief inverse yields the employment multipliers ( $\lambda$ ):

$$(4) \quad \lambda = l(I - A)^{-1}.$$

In traditional input-output analysis all matrices are square ( $m = n$ ) and equations (1) and (3) imply the well-known commodity technology coefficients  $A = U(V^T)^{-1}$  and  $l = L(V^T)^{-1}$  (Kop Jansen and ten Raa, 1990). In this case, the employment multipliers (4) reduce to:

$$(5) \quad \lambda L(V^T)^{-1} [I - U(V^T)^{-1}]^{-1} = L\{[I - U(V^T)^{-1}]V^T\}^{-1} = L(V^T - U)^{-1}.$$

or

$$(6) \quad L = \lambda(V^T - U).$$

If there are more activities than commodities ( $m > n$ ), the system of equations (6) is overdetermined, an error term must be attached, and the employment multipliers become regression coefficients:

$$(7) \quad L = \lambda(V^T - U) + \varepsilon,$$

In (7)  $L$  is a row vector of order  $m$  with labor employment,  $\lambda$  is a row vector of order  $n$  with employment multipliers,  $V$  is a make matrix of order  $m \times n$ ,  $U$  is a use matrix of order  $n \times m$  and  $\varepsilon$  is a row vector of independently normally random

disturbance errors with zero mean and constant variance, with order  $m$ . Notice that  $m$  is the number of establishments or the observations. The estimation of employment multipliers becomes a matter of multiple linear regression analysis, with linear, unbiased and consistent multipliers estimates. In section 4, we estimate (7) for the Andalusian economy in the year 1995.

### *Output multipliers*

Output multipliers,  $\mu$  are given by the column totals of the Leontief inverse:

$$(8) \quad \mu = e^T(I - A)^{-1}$$

The only difference with equation (4) is the replacement of the row vector of labor coefficients  $l$  by the unit vector  $e = (1 \dots 1)$ . In traditional input-output analysis the output multipliers (8) reduce to:

$$(9) \quad \mu = e [I - U(V^T)^{-1}]^{-1} = eV^T(V^T - U)^{-1}$$

or

$$(10) \quad eV^T = \mu(V^T - U).$$

If there are more activities than commodities ( $m > n$ ), the system of equations (10) is overdetermined, an error term must be attached, and the output multipliers become regression coefficients:

$$(11) \quad eV^T = \mu(V^T - U) + \varepsilon,$$

In (11)  $eV^T$  is the row vector of total outputs of establishments (of order  $m$ ),  $\mu$  is a row vector of output multipliers (of order  $n$ ),  $V$  make matrix of order  $m \times n$ ,  $U$  is the use matrix of order  $n \times m$  and  $\varepsilon$  is a row vector of independently normally random disturbance errors with zero mean and constant variance, with order  $m$ . In section 4, we estimate (11) for the Andalusian economy in the year 1995. The huge sample size justifies our normality assumption (by the Central Limit Theorem).

## **4. RESULTS**

We use a sample of 18,084 observations; it covers nearly 45% of the Andalusian production and more than a third of employment. Most observations are

establishments, but for some industries data had to be consolidated into single observations (see Table 1). The Institute of Statistics of Andalusia (IEA, regional statistical office) has adjusted the sample (using other information from different statistical sources and/or companies) to produce the mainly survey-based Andalusian Input-Output Framework 1995. The sampling of the different surveys conducted was done on an industry by employment basis. For each of a given set of employment brackets, predetermined rules were applied to select establishments from the full population. The largest establishments (in terms of number of employees) were all selected, while smaller establishments were selected by a stratified simple random sample. We refer to IEA (1999) for a detailed description of the whole procedure and sample coverage.

#### *Employment multipliers*

The employment multiplier estimates are presented in Table 1. For comparison, the second column displays the employment multipliers based on published use and make matrices.

The model has been estimated for 87 commodities by means of ordinary least squares. Due to the presence of certain forms of unknown heteroscedasticity, the White estimate (White, 1980) is used for estimating the covariance matrix of estimated coefficients, which provide consistent standard errors. We find that problems of autocorrelation and multicollinearity do not plague our analysis. Only 12 out of 7.482 possible correlations were higher than 0.5 with only one higher than 0.75. 76 estimated multipliers are significant at the 95% confidence level and the R-squared is 0.9948, which is also satisfactory.

Two main contributions are provided by the results presented in the Table 1. They are: (a) in most cases, published use and make matrices based employment multipliers *overestimate*, contrary to Simonovits (1975) theoretical result and (b) the bias of employment multipliers is generally positively related with the secondary products of an economy.

**TABLE 1**  
**EMPLOYMENT MULTIPLIERS (NUMBER OF WORKERS PER 600.000 EUROS)**

	<b>Description</b>	<b>Sample size</b>	<b>Estimated multiplier</b>	<b>Multiplier (MIOAN95)</b>	<b>p value</b>	<b>Lower bound</b>	<b>Upper bound</b>
89	Household employers services	1	116.2	116.2	0	116.2	116.2
83	Private social services	1	40.2	40.6	0	39.2	41.2
88	Personal services	1	31.9	32.6	0	31.6	32.3
75	Cleaning services	1	32.7	32.1	0	31.6	33.7
74	Security services	48	23.5	31.6	0.0004	10.5	36.4
5	Forestry and related services	1	30	30.3	0	29.5	30.5
79	Private education services	150	31.8	28.1	0	29.8	33.8
82	Public social services	1	25.6	25.7	0	25.4	25.9
1	Fruits and vegetables	1	23.9	24.1	0	23.3	24.6
44	Furniture	601	9.7	23.1	0.0017	3.7	15.8
24	Cork and wood products	299	8	22.6	0.0413	0.3	15.7
55	Retail trade and repair domestic and personal effects	4,113	42.9	22.1	0.1186	-11	96.8
73	Marketing services	8	1.5	21.5	0.3509	-1.7	4.7
4	Livestock and hunting	1	21.1	21.3	0	20.7	21.5
2	Olive and vine	1	21	21.2	0	20.5	21.5
70	Research and development	1	20.1	19.9	0	19.5	20.6
76	Other business services	1	4.3	19.7	0.7309	-20.3	29
57	Bars and restaurants services	352	31.1	18.8	0	23.2	39.1
85	Social services	1	22.2	18.8	0	14.2	30.2
77	Public Administration	1	19	18.6	0	18.2	19.8
23	Leather tanning, leather products and footwear	135	7.2	18.5	0.0013	2.8	11.5
22	Clothing products	483	4.4	18.4	0.0186	0.7	8
51	Preparing, installation and finishing construction services	1	18.3	18.4	0	17.7	18.8
6	Fish and fishing products	173	11	18.3	0	9.7	12.4
71	Accounting and law activity services	235	72	18.1	0.106	-15.3	159.3
32	Ceramics, clay, bricks and other products for building	126	11.7	17.7	0	8.6	14.7
52	Petrol and motor vehicles trade services	380	18.1	17.6	0.1137	-4.3	40.4
13	Canned and preserved fish, fruit and vegetables	148	15	17.5	0	12.1	17.9
78	Public education services	2	18.3	17.5	0	17.4	19.2
59	Other earthbound transportation services	1	17	17.1	0	16.1	17.8
80	Public medical and hospitals services	1	16.6	16.8	0	16.4	16.8
72	Engineering and architecture technical services	31	23.2	16.5	0.0203	3.6	42.8
84	Public drainage and sewerage services	15	23.8	16.5	0	20.5	27.1
53	Repair motor vehicles services	1	15.9	16.3	0	15.5	16.3
65	Insurance	1	12.1	15.9	0	10.3	13.9
16	Grain mills, bakery, sugar mills, ...	469	4.6	15.6	0	2.7	6.5
35	Fabricated metal products	1,087	6.6	15.6	0.0001	3.3	9.9
33	Stone and glass products	282	7.1	15.5	0	5.2	9.1
42	Naval transportation and repairing services	65	2.3	15.4	0.1319	-0.7	5.4
66	Allied financial services	1	6.4	15.4	0.0021	2.3	10.5
3	Other agriculture and related services	1	14.6	14.8	0	14.2	15.1
45	Miscellaneous manufactured products	1	14.6	14.6	0	13.9	15.3
14	Fats and oils	252	3.4	14.5	0	2.2	4.7

**TABLE 1 (Continued)**  
**EMPLOYMENT MULTIPLIERS (NUMBER OF WORKERS PER 600.000 EUROS)**

Description	Sample size	Estimated multiplier	Multiplier (MIOAN95)	p value	Lower bound	Upper bound
56 Hotels services	20	19.4	14.1	0	12.8	26
36 Machinery and mechanic equipment	1	14.1	14	0	13.4	14.8
7 Coal mining	4	7.2	13.7	0	5	9.3
60 Sea and river transportation services	2	7.2	13.7	0	5	9.4
62 Allied transportation services	142	8.1	13.6	0.0007	3.4	12.8
69 Computer services	84	15.1	12.7	0.3185	-14.6	44.9
54 Wholesale trade	4,535	3.4	12.4	0.3184	-3.3	10.2
87 Other amusement, cultural, sport and recreation services	538	9.6	12.4	0	8.4	10.9
50 Constructions	1,136	7	12.2	0	5.2	8.7
49 Water and sewerage services	67	10.8	11.8	0	8.1	13.5
18 Wines and alcoholic beverages	105	7.7	11.7	0	6.2	9.3
15 Milk and dairy products	34	11.5	11.6	0	6.5	16.4
46 Recycling products	1	9.8	11.1	0	8	11.5
58 Railway transportation services	1	10.6	11.1	0	9.9	11.2
68 Machinery and equipment rental	9	8.8	11	0.0001	4.5	13.2
11 Non-metallic and non-energetic minerals	1	11	10.9	0	10.1	12
26 Printing, publishing and editing services	447	33.9	10.7	0.0868	-4.9	72.7
21 Textile mill products	1	10.6	10.5	0	10.2	11
43 Miscellaneous transportation equipment	22	1.5	10.5	0.2899	-1.2	4.2
63 Post and communications services	205	5.2	10.4	0	4.7	5.8
31 Cement, lime and allied products	200	4.3	10.2	0.0018	1.6	7
12 Meat and meat products	239	6.4	9.8	0.0001	3.2	9.7
40 Professional and scientific instruments	60	3.6	9.2	0.0664	-0.2	7.4
30 Rubber and plastic products	122	3.3	9.1	0.0001	1.6	4.9
64 Finances	1	8.7	9.1	0	7.9	9.5
10 Metallic minerals	7	7	8.8	0	4.2	9.8
29 Other chemical products	96	5	8.7	0.0185	0.8	9.2
38 Electrical and electronic machinery	88	3.6	8.6	0.0166	0.7	6.6
41 Motor vehicles transportation equipment	45	11.3	8.6	0	6.4	16.2
61 Air transportation services	1	8.4	7.6	0	4.7	12.1
81 Private medical and hospitals services	4	4.9	7.6	0.0008	2	7.8
39 Electronic materials, radio and television equipments	1	7.7	7.5	0	6.9	8.5
19 Beer and soft drinks	23	5.5	7.2	0	4.7	6.3
25 Paper and allied products	51	3.6	7.1	0	2	5.1
20 Tobacco products	3	5.4	7	0.0478	0.1	10.7
17 Miscellaneous food products	139	6	6.8	0.001	2.4	9.5
37 Computers and office equipments	1	5.5	5.9	0	4.7	6.3
86 Cinema, video, radio and television services	1	29.1	5.9	0	25.2	33
28 Basic chemical products	52	2.2	5.1	0.0079	0.6	3.7
48 Gas and water steam and irrigation services	1	3.5	3.8	0	2.1	5
47 Electricity and irrigations services	51	2.8	3.4	0	2.6	2.9
34 Primary metal products	58	0.8	2.3	0.005	0.2	1.3
67 Real Estate	1	2.1	2	0	1.6	2.6
27 Petroleum refining products	9	2	1.6	0	1.5	2.6

Source: Own elaboration.

Indeed, 57 out of 87 commodities have lower employment multipliers than those calculated with published use and make matrices. On the contrary, 19 commodities have higher employment multipliers. Our findings may contradict the underestimation of the Leontief inverse found in the theoretical literature, see inequality (2).

Since our estimated values are unbiased and consistent, it seems that most of the employment multipliers obtained by using published data are overestimated and not underestimated, confirming the intuition of Dietzenbacher (1995) and Roland-Holst (1989). Simonovits (1975) results rest upon some restrictive conditions (e.g. independence of technical coefficients) that we have not assumed at all.

Table 2 confirms that industries with sizeable secondary production and commodities of which most output is secondary product, have employment multipliers with a large bias, as measured by the difference between published-data based and estimated employment multipliers. (The linear correlation coefficient between the bias in absolute values and the sum of columns 3 and 4 is 0.45). Estimated multipliers not significant at a 95% confidence level are set zero, so that the bias equals the official data based multiplier. The same will be done for output multipliers later on.

From a theoretical view, when some industries with no secondary activities produce commodities for which other industries provide sizeable amounts, it is reasonable to assume that the technologies used by the rest of the economy for making such commodities should not match that of the industries for which they are primary products (actually, the latter industries' technology can be considered as commodities technology since no secondary products are produced). This could explain the sizeable bias of marketing services, computer services and public drainage and sewerage services.

On the contrary, when some commodities are produced by a single industry with large proportions of secondary outputs, it is reasonable to assume that these primary commodities are not produced according to a commodity technology hypothesis if the bias is sizeable.

Such are the cases of cinema, video, radio and television services and printing, publishing and editing services.

Other business services have large proportions of secondary activities (42.74%) and their primary products are produced elsewhere in sizeable amounts (41.45%).

**TABLE 2**  
**SECONDARY PRODUCTS SHARES OVER TOTAL INDUSTRIES AND/OR COMMODITIES**  
**PRODUCTIONS AND BIAS OF EMPLOYMENT MULTIPLIERS**

	Description	Over industry (%)	Over product (%)	Bias (absolute values)
73	Marketing services	3.17	81.91	21.49
76	Other business services	42.74	41.45	19.68
69	Computer services	12.77	40.30	12.75
86	Cinema, video, radio and television services	43.97	3.59	23.17
87	Other amusement, cultural, sport and recreation services	35.32	4.11	2.78
84	Public drainage and sewerage services	1.66	36.41	7.23
71	Accounting and law activity services	12.83	19.88	18.09
26	Printing, publishing and editing services	28.24	1.68	10.74
56	Hotels services	26.33	3.31	5.28
10	Metallic minerals	28.18	0.00	1.86
15	Milk and dairy products	5.63	21.72	0.10
85	Social services	25.80	0.00	3.36
6	Fish and fishing products	24.74	0.01	7.29
49	Water and sewerage services	6.53	18.19	0.98
61	Air transportation services	19.85	0.00	0.81
29	Other chemical products	15.43	3.64	3.69
3	Other agriculture and related services	2.87	15.94	0.25
74	Security services	1.61	16.71	8.16
17	Miscellaneous food products	12.62	4.77	0.84
59	Other earthbound transportation services	1.66	15.56	0.11
13	Canned and preserved fish, fruit and vegetables	3.80	12.54	2.55
52	Petrol and motor vehicles trade services	14.79	0.04	17.56
5	Forestry and related services	6.00	7.65	0.32
51	Preparing, installation and finishing construction services	9.43	4.12	0.12
79	Private education services	10.46	1.20	3.65
4	Livestock and hunting	1.42	10.17	0.24
12	Meat and meat products	7.98	2.35	3.32
28	Basic chemical products	2.19	7.20	2.90
72	Engineering and architecture technical services	0.52	8.85	6.63
57	Bars and restaurants services	1.20	7.49	12.25
18	Wines and alcoholic beverages	7.76	0.88	3.95
53	Repair motor vehicles services	1.97	6.40	0.46
62	Allied transportation services	4.41	3.22	5.53
77	Public Administration	7.62	0.00	0.35
16	Grain mills, bakery, sugar mills, ...	3.31	4.00	11.01
63	Post and communications services	1.05	5.17	5.20
36	Machinery and mechanic equipment	4.20	1.85	0.12
30	Rubber and plastic products	3.21	2.43	5.79
11	Non-metallic and non-energetic minerals	3.44	2.19	0.12
67	Real Estate	0.87	4.71	0.11
21	Textile mill products	4.13	1.42	0.09
37	Computers and office equipments	0.21	5.01	0.42
2	Olive and vine	5.08	0.11	0.21
50	Constructions	1.78	3.42	5.20

**TABLE 2 (Continued)**  
**SECONDARY PRODUCTS SHARES OVER TOTAL INDUSTRIES AND/OR COMMODITIES**  
**PRODUCTIONS AND BIAS OF EMPLOYMENT MULTIPLIERS**

	Description	Over industry (%)	Over product (%)	Bias (absolute values)
55	Retail trade and repair domestic and personal effects	4.54	0.64	22.12
54	Wholesale trade	1.19	3.88	12.42
68	Machinery and equipment rental	3.00	2.05	2.21
25	Paper and allied products	2.54	2.33	3.57
35	Fabricated metal products	1.18	3.59	9.04
14	Fats and oils	3.70	1.03	11.03
88	Personal services	0.35	3.97	0.78
83	Private social services	2.82	1.49	0.46
32	Ceramics, clay, bricks and other products for building	1.49	2.57	5.99
19	Beer and soft drinks	0.71	3.28	1.72
38	Electrical and electronic machinery	0.75	3.08	4.93
24	Cork and wood products	2.81	0.97	14.63
7	Coal mining	3.58	0.00	6.54
23	Leather tanning, leather products and footwear	2.88	0.52	11.29
27	Petroleum refining products	2.86	0.48	0.47
44	Furniture	0.54	2.71	13.40
1	Fruits and vegetables	3.01	0.21	0.25
31	Cement, lime and allied products	2.07	1.13	5.91
45	Miscellaneous manufactured products	1.64	1.51	0.01
39	Electronic materials, radio and television equipments	3.13	0.02	0.24
40	Professional and scientific instruments	1.09	1.77	9.18
43	Miscellaneous transportation equipment	2.57	0.19	10.52
33	Stone and glass products	1.24	1.40	8.33
58	Railway transportation services	2.58	0.00	0.48
42	Naval transportation and repairing services	2.33	0.22	15.37
47	Electricity and irrigations services	1.82	0.57	0.65
34	Primary metal products	2.13	0.23	1.56
41	Motor vehicles transportation equipment	1.43	0.37	2.66
75	Cleaning services	1.37	0.28	0.51
22	Clothing products	1.42	0.16	14.06
48	Gas and water steam and irrigation services	1.11	0.21	0.24
66	Allied financial services	0.05	0.65	9.02
81	Private medical and hospitals services	0.32	0.09	2.73
78	Public education services	0.18	0.00	0.79
70	Research and development	0.04	0.00	0.12
60	Tobacco products	0.00	0.00	6.54
65	Recycling products	0.00	0.00	3.80
20	Sea and river transportation services	0.00	0.00	1.64
46	Finances	0.00	0.00	1.39
64	Insurance	0.00	0.00	0.42
89	Public medical and hospitals services	0.00	0.00	0.38
80	Public social services	0.00	0.00	0.20
82	Household employers services	0.00	0.00	0.14

Source: Own elaboration.

### *Output multipliers*

Maintaining the number of observations of the last section, the output multipliers are as presented in Table 3.

The model has been estimated for 87 commodities by means of ordinary least squares. As before, the White estimate (White, 1980) is used for estimating the covariance matrix of estimated coefficients, which provide consistent standard errors. Autocorrelation and multicollinearity problems do not plague our analysis. 84 estimated multipliers are significant at the 95% confidence level and the R-squared is 0.9993.

The output multipliers results presented above provide similar employment multipliers contributions. That is: (a) mostly, published use and make matrices based output multipliers are *overestimated* and not underestimated; and (b) in general, the bias of output multipliers has a positive relationship with economy-wide secondary products. We dwell on these points.

Firstly, it is remarkable that 73 out of 87 commodities have lower output multipliers than published data based multipliers. On the contrary, 11 commodities have higher output multipliers. Recall that since Simonovits (1975) results rest upon some restrictive conditions (e.g. independence of technical coefficients) that we have not assumed at all, our findings may not contradict the underestimation of the Leontief inverse found in the theoretical literature. Since our estimated values are unbiased and consistent, it seems that most of the output multipliers obtained by using published data are overestimated and not underestimated, confirming the intuition of Dietzenbacher (1995) and Roland-Holst (1989).

Secondly, as shown in Table 4, those industries with relevant secondary productions and those commodities for which most of their outputs are secondary product, have output multipliers with larger bias, measured by the difference between published-data based and estimated output multipliers. (The linear correlation coefficient between the bias in absolute values and the sum of columns 3 and 4 is 0.3).

**TABLE 3**  
**OUTPUT MULTIPLIERS**

	<b>Description</b>	<b>Estimated multiplier</b>	<b>Multiplier (MIOAN95)</b>	<b>p value</b>	<b>Lower bound</b>	<b>Upper bound</b>
60	Sea and river transportation services	1.876	2.256	0.0000	1.731	2.021
85	Social services	2.136	2.124	0.0000	1.911	2.361
14	Fats and oils	1.439	1.949	0.0000	1.335	1.544
13	Canned and preserved fish, fruit and vegetables	1.755	1.872	0.0000	1.557	1.952
7	Coal mining	0.724	1.799	0.0325	0.060	1.388
65	Insurance	1.711	1.740	0.0000	1.658	1.764
31	Cement, lime and allied products	1.183	1.739	0.0000	1.001	1.365
18	Wines and alcoholic beverages	1.433	1.696	0.0000	1.349	1.518
73	Marketing services	1.046	1.680	0.0000	0.923	1.170
50	Constructions	1.353	1.670	0.0000	1.240	1.467
12	Meat and meat products	1.447	1.664	0.0000	1.240	1.654
28	Basic chemical products	1.206	1.621	0.0000	1.049	1.363
62	Allied transportation services	1.106	1.607	0.0000	1.031	1.181
15	Milk and dairy products	1.654	1.603	0.0000	1.380	1.929
11	Non-metallic and non-energetic minerals	1.523	1.562	0.0000	1.491	1.555
33	Stone and glass products	1.299	1.546	0.0000	1.182	1.415
16	Grain mills, bakery, sugar mills, ...	1.201	1.534	0.0000	1.070	1.333
83	Private social services	1.471	1.524	0.0000	1.442	1.499
57	Bars and restaurants services	1.355	1.523	0.0000	1.204	1.505
42	Naval transportation and repairing services	1.024	1.502	0.0000	0.999	1.049
46	Recycling products	1.346	1.493	0.0000	1.214	1.477
71	Accounting and law activity services	2.465	1.477	0.0690	-0.191	5.121
59	Other earthbound transportation services	1.422	1.465	0.0000	1.406	1.438
44	Furniture	1.210	1.464	0.0000	1.070	1.350
87	Other amusement, cultural, sport and recreation services	1.689	1.457	0.0000	1.651	1.727
49	Water and sewerage services	1.293	1.448	0.0000	1.100	1.486
88	Personal services	1.385	1.443	0.0000	1.370	1.400
47	Electricity and irrigations services	1.072	1.434	0.0000	1.064	1.080
24	Cork and wood products	0.679	1.431	0.0069	0.186	1.171
66	Allied financial services	1.421	1.431	0.0000	1.302	1.541
76	Other business services	1.002	1.427	0.0300	0.097	1.906
19	Beer and soft drinks	1.309	1.417	0.0000	1.204	1.414
32	Ceramics, clay, bricks and other products for building	1.182	1.415	0.0000	1.089	1.275
86	Cinema, video, radio and television services	2.097	1.411	0.0000	1.937	2.256
72	Engineering and architecture technical services	1.156	1.409	0.0000	0.784	1.528
17	Miscellaneous food products	1.300	1.402	0.0000	1.070	1.530
55	Retail trade and repair domestic and personal effects	0.917	1.391	0.0000	0.785	1.049
51	Preparing, installation and finishing construction services	1.352	1.374	0.0000	1.336	1.368
45	Miscellaneous manufactured products	1.340	1.367	0.0000	1.319	1.362
23	Leather tanning, leather products and footwear	1.177	1.366	0.0000	1.070	1.284
10	Metallic minerals	1.337	1.365	0.0000	1.291	1.383
4	Livestock and hunting	1.325	1.357	0.0000	1.307	1.343
37	Computers and office equipments	1.296	1.352	0.0000	1.281	1.310
84	Public drainage and sewerage services	1.165	1.346	0.0000	1.095	1.235
22	Clothing products	1.165	1.336	0.0000	0.969	1.361
56	Hotels services	1.264	1.329	0.0000	1.146	1.383

TABLE 3 (Continued)  
OUTPUT MULTIPLIERS

	Description	Estimated multiplier	Multiplier (MIOAN95)	p value	Lower bound	Upper bound
1	Fruits and vegetables	1.266	1.323	0.0000	1.253	1.280
77	Public Administration	1.309	1.317	0.0000	1.286	1.332
25	Paper and allied products	1.117	1.314	0.0000	1.040	1.193
61	Air transportation services	1.321	1.304	0.0000	1.180	1.461
52	Petrol and motor vehicles trade services	-0.168	1.295	0.5752	-0.757	0.420
30	Rubber and plastic products	0.921	1.290	0.0000	0.760	1.082
21	Textile mill products	1.273	1.289	0.0000	1.260	1.286
29	Other chemical products	1.031	1.287	0.0000	0.959	1.103
79	Private education services	1.193	1.284	0.0000	1.141	1.245
36	Machinery and mechanic equipment	1.266	1.284	0.0000	1.241	1.290
35	Fabricated metal products	1.040	1.272	0.0000	0.928	1.151
3	Other agriculture and related services	1.226	1.264	0.0000	1.216	1.235
5	Forestry and related services	1.224	1.262	0.0000	1.212	1.235
54	Wholesale trade	0.593	1.253	0.0000	0.345	0.840
58	Railway transportation services	1.220	1.252	0.0000	1.202	1.238
27	Petroleum refining products	1.271	1.245	0.0000	1.233	1.309
26	Printing, publishing and editing services	0.507	1.243	0.1699	-0.217	1.231
41	Motor vehicles transportation equipment	1.318	1.243	0.0000	1.142	1.494
68	Machinery and equipment rental	1.151	1.236	0.0000	1.053	1.250
53	Repair motor vehicles services	1.197	1.232	0.0000	1.186	1.207
43	Miscellaneous transportation equipment	1.032	1.232	0.0000	1.000	1.065
69	Computer services	1.597	1.231	0.0001	0.796	2.398
6	Fish and fishing products	1.163	1.230	0.0000	1.003	1.323
81	Private medical and hospitals services	1.214	1.208	0.0000	1.169	1.259
39	Electronic materials, radio and television equipments	1.170	1.202	0.0000	1.154	1.186
34	Primary metal products	1.058	1.200	0.0000	1.015	1.101
82	Public social services	1.178	1.195	0.0000	1.171	1.185
64	Finances	1.183	1.193	0.0000	1.153	1.212
2	Olive and vine	1.142	1.179	0.0000	1.133	1.151
20	Tobacco products	1.117	1.159	0.0000	0.958	1.276
38	Electrical and electronic machinery	1.045	1.156	0.0000	1.007	1.083
40	Professional and scientific instruments	1.043	1.155	0.0000	0.975	1.111
48	Gas and water steam and irrigation services	1.050	1.147	0.0000	0.855	1.245
63	Post and communications services	1.069	1.145	0.0000	1.062	1.075
80	Public medical and hospitals services	1.130	1.142	0.0000	1.126	1.135
75	Cleaning services	1.104	1.110	0.0000	1.078	1.129
74	Security services	1.227	1.094	0.0000	0.801	1.654
67	Real Estate	1.082	1.088	0.0000	1.066	1.097
70	Research and development	1.076	1.086	0.0000	1.049	1.102
78	Public education services	1.027	1.051	0.0000	1.009	1.045
89	Household employers services	1.000	1.000	0.0000	1.000	1.000

Source: Own elaboration.

**TABLE 4**  
**SECONDARY PRODUCTS SHARES OVER TOTAL INDUSTRIES AND/OR COMMODITIES**  
**PRODUCTIONS AND BIAS OF OUTPUT MULTIPLIERS**

	Description	Over industry (%)	Over product (%)	Bias (absolute values)
73	Marketing services	3.17	81.91	0.634
76	Other business services	42.74	41.45	0.425
69	Computer services	12.77	40.30	0.366
86	Cinema, video, radio and television services	43.97	3.59	0.686
87	Other amusement, cultural, sport and recreation services	35.32	4.11	0.233
84	Public drainage and sewerage services	1.66	36.41	0.181
71	Accounting and law activity services	12.83	19.88	1.477
26	Printing, publishing and editing services	28.24	1.68	1.243
56	Hotels services	26.33	3.31	0.064
10	Metallic minerals	28.18	0.00	0.028
15	Milk and dairy products	5.63	21.72	0.051
85	Social services	25.80	0.00	0.012
6	Fish and fishing products	24.74	0.01	0.067
49	Water and sewerage services	6.53	18.19	0.155
61	Air transportation services	19.85	0.00	0.016
29	Other chemical products	15.43	3.64	0.256
3	Other agriculture and related services	2.87	15.94	0.039
74	Security services	1.61	16.71	0.134
17	Miscellaneous food products	12.62	4.77	0.102
59	Other earthbound transportation services	1.66	15.56	0.043
13	Canned and preserved fish, fruit and vegetables	3.80	12.54	0.118
52	Petrol and motor vehicles trade services	14.79	0.04	1.295
5	Forestry and related services	6.00	7.65	0.039
51	Preparing, installation and finishing construction services	9.43	4.12	0.022
79	Private education services	10.46	1.20	0.091
4	Livestock and hunting	1.42	10.17	0.032
12	Meat and meat products	7.98	2.35	0.217
28	Basic chemical products	2.19	7.20	0.414
72	Engineering and architecture technical services	0.52	8.85	0.253
57	Bars and restaurants services	1.20	7.49	0.169
18	Wines and alcoholic beverages	7.76	0.88	0.263
53	Repair motor vehicles services	1.97	6.40	0.035
62	Allied transportation services	4.41	3.22	0.501
77	Public Administration	7.62	0.00	0.008
16	Grain mills, bakery, sugar mills, ...	3.31	4.00	0.333
63	Post and communications services	1.05	5.17	0.076
36	Machinery and mechanic equipment	4.20	1.85	0.018
30	Rubber and plastic products	3.21	2.43	0.369
11	Non-metallic and non-energetic minerals	3.44	2.19	0.039
67	Real Estate	0.87	4.71	0.007
21	Textile mill products	4.13	1.42	0.016
37	Computers and office equipments	0.21	5.01	0.057
2	Olive and vine	5.08	0.11	0.037
50	Constructions	1.78	3.42	0.317

**TABLE 4 (Continued)**  
**SECONDARY PRODUCTS SHARES OVER TOTAL INDUSTRIES AND/OR COMMODITIES**  
**PRODUCTIONS AND BIAS OF OUTPUT MULTIPLIERS**

	Description	Over industry (%)	Over product (%)	Bias (absolute values)
55	Retail trade and repair domestic and personal effects	4.54	0.64	0.474
54	Wholesale trade	1.19	3.88	0.660
68	Machinery and equipment rental	3.00	2.05	0.084
25	Paper and allied products	2.54	2.33	0.197
35	Fabricated metal products	1.18	3.59	0.232
14	Fats and oils	3.70	1.03	0.509
88	Personal services	0.35	3.97	0.058
83	Private social services	2.82	1.49	0.053
32	Ceramics, clay, bricks and other products for building	1.49	2.57	0.234
19	Beer and soft drinks	0.71	3.28	0.108
38	Electrical and electronic machinery	0.75	3.08	0.111
24	Cork and wood products	2.81	0.97	0.752
7	Coal mining	3.58	0.00	1.075
23	Leather tanning, leather products and footwear	2.88	0.52	0.190
27	Petroleum refining products	2.86	0.48	0.026
44	Furniture	0.54	2.71	0.254
1	Fruits and vegetables	3.01	0.21	0.057
31	Cement, lime and allied products	2.07	1.13	0.557
45	Miscellaneous manufactured products	1.64	1.51	0.027
39	Electronic materials, radio and television equipments	3.13	0.02	0.032
40	Professional and scientific instruments	1.09	1.77	0.112
43	Miscellaneous transportation equipment	2.57	0.19	0.199
33	Stone and glass products	1.24	1.40	0.247
58	Railway transportation services	2.58	0.00	0.032
42	Naval transportation and repairing services	2.33	0.22	0.478
47	Electricity and irrigations services	1.82	0.57	0.362
34	Primary metal products	2.13	0.23	0.142
41	Motor vehicles transportation equipment	1.43	0.37	0.075
75	Cleaning services	1.37	0.28	0.006
22	Clothing products	1.42	0.16	0.171
48	Gas and water steam and irrigation services	1.11	0.21	0.097
66	Allied financial services	0.05	0.65	0.010
81	Private medical and hospitals services	0.32	0.09	0.006
78	Public education services	0.18	0.00	0.024
70	Research and development	0.04	0.00	0.011
60	Tobacco products	0.00	0.00	0.380
46	Recycling products	0.00	0.00	0.148
20	Sea and river transportation services	0.00	0.00	0.042
65	Finances	0.00	0.00	0.029
82	Insurance	0.00	0.00	0.017
80	Public medical and hospitals services	0.00	0.00	0.012
64	Public social services	0.00	0.00	0.010
89	Household employers services	0.00	0.00	0.000

Source: Own elaboration.

## 5. CONCLUSIONS

Technical coefficients are the subject of two disjunct bodies of literature. The construction of technical coefficients is linked to flow data (use and make matrices), but stochastics are imposed on the coefficients when multipliers are calculated, by means of the Leontief inverse. Due the nonlinearity of this operation, the multiplier estimates are biased (it is generally argued that the Leontief inverse underestimates input-output multipliers).

In this paper, we let the flow data tell the stochastics and take them all the way to confidence intervals for multipliers. We focus on the use and make matrices instead of the  $A$ -matrix to obtain unbiased and consistent multipliers estimates. Our output and employment multipliers are normally distributed and do not suffer from over- or underestimation. Our results for the Andalusian economy indicate that the Leontief inverse is not underestimated but overestimated in most of the cases.

Statistical offices combine use and make flow data (including inversion of the make matrix) to construct input-output coefficients and economists invert the Leontief matrix to determine the output and cost multipliers of the economy. The construction and the inversion are nonlinear operations with complicated errors transmission and have been studied in relative isolation. This paper shows, however, that a shortcut from the use and make data to the multipliers provides simple, unbiased and consistent estimates.

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## APPENDIX: DATA

The Andalusian Input-Output Framework 1995 (MIOAN95) is one of the first Spanish input-output tables based on the new European System of Accounts (ESA-95) published by EUROSTAT (1996). The Institute of Statistics of Andalusia (IEA, regional statistical office) provided the cross-section inputs and outputs establishment data. These data were used for the elaboration of the Input-Output Andalusian Framework 1995 (IEA, 1999).

The Institute of Statistics of Andalusia (IEA) publishes two use tables, which differ by valuation. One is valued at purchasers' prices and the other at basic prices, which is the same as the former but excluding trade and transport margins and net commodity taxes; see Viet (1994, p.28). (Trade and transport margins are simply reallocated from the commodities where they are included, at purchasers' values, to the use matrix rows of trade and transport services). The make table is published exclusively at basic prices. The United Nations System of National Accounts (SNA) recommends basic values; production costs of good and services are measured before they are conveyed to the market for consumption so that the effects of tax and subsidy policies as well as of differences in types of economic transactions are isolated.

IEA transforms a use data at purchasers' prices into basic prices, as described below. The use and make tables at basic prices are balanced to obtain the final official accounts for the Input-Output framework.

Since all input and output data provided by IEA were valued at purchasers' prices and at basic prices, respectively, we subtracted trade and transport margins and also net commodity taxes from establishment inputs in order to have the same valuation (basic prices) for inputs and outputs and to estimate equations (7) and (10). As detailed below, we applied the same formula as IEA used for the elaboration of the use matrix at basic prices and assumed equality of margins and net commodity taxes between establishments in industry  $j$ , which consume some commodity  $k$ . We will focus now on the procedure.

According to the ESA-95, the intermediate uses at basic values are equal to the intermediate uses at purchasers' prices minus trade and transport margins and minus net commodity taxes. Let  $u_{kj}^b$  and  $u_{kj}^p$  be the total domestic inputs of commodity  $k$  by industry  $j$  at basic and at purchasers' prices, respectively. Then, we can write out that:

$$(12) \quad u_{kj}^b = u_{kj}^p - T_{kj}^d - T_{kj} - N_{kj} - H_{kj},$$

where, for each use of commodity  $k$  by industry  $j$ ,  $T_{kj}^d$  and  $T_{kj}$  are the total amount of trade and transport margins, respectively,  $N_{kj}$  is the total amount of net commodity taxes (excluding not deductible value added tax, VAT) and  $H_{kj}$  is the total amount of not deductible value added tax.

We assume that the trade margins are proportional to the use data at purchasers' prices. The proportions are defined by:

$$(13) \quad T_{kj}^d = t_{kj}^d u_{kj}^p, \quad 0 < t_{kj}^d < 1.$$

We assume now that net commodity taxes (excluding not deductible VAT) and transport margins are proportional to the use data at basic prices:

$$(14) \quad N_{kj} = n_{kj} u_{kj}^b, \quad 0 < n_{kj} < 1;$$

$$(15) \quad T_{kj} = t_{kj} u_{kj}^b, \quad 0 < t_{kj} < 1.$$

With respect to the value added tax, the assumption is as follows:

$$(16) \quad H_{kj} = h_{kj} \left( \frac{u_{kj}^p}{1 + h_{kj}} \right), \quad 0 < h_{kj} < 1.$$

Substituting (13), (14), (15) and (16) in (12), we obtain:

$$(17) \quad u_{kj}^b = u_{kj}^p \left( \frac{1 - t_{kj}^d - \frac{h_{kj}}{1 + h_{kj}}}{1 + t_{kj} + n_{kj}} \right),$$

This formula is used to transform use data from basic values to purchasers' values.

Our purpose is to estimate the unknown  $u_{kji}^b$ , that is, the total use of commodity  $k$  by an establishment  $i$  from industry  $j$  at basic prices. Then, since survey available data is based on establishments of a particular industry and not on products, we denote  $u_{kji}^p$  and  $u_{kji}^b$  as purchasers' and basic prices use data, respectively. Based on (17), our objective would be to apply the following formula for each establishment,  $i$ :

$$(18) \quad u_{kji}^b = u_{kji}^p \left( \frac{1 - t_{kji}^d - \frac{h_{kji}}{1 + h_{kji}}}{1 + t_{kji} + n_{kji}} \right).$$

A problem arises when available information does not enable us to value establishment specific  $t_{kji}^d$ ,  $h_{kji}$ ,  $t_{kji}$  and  $n_{kji}$ . In this case, we assume equality of margins and net commodity taxes across firms in industry  $j$ , which consume some commodity  $k$ . We consequently use (17) with:

$$\begin{aligned} t_{kji}^d &= t_{kj}^d && \text{for all } i, \\ t_{kji} &= t_{kj} && \text{for all } i, \\ n_{kji} &= n_{kj} && \text{for all } i, \\ h_{kji} &= h_{kj} && \text{for all } i, \end{aligned}$$

so that the formula becomes:

$$(18) \quad u_{kji}^b = u_{kji}^p \left( \frac{1 - t_{kj}^d - \frac{h_{kj}}{1 + h_{kj}}}{1 + t_{kj} + n_{kj}} \right).$$

Once trade and transport margins and net commodity taxes have been subtracted from use flow data, the last step is to allocate the subtracted total trade and domestic transport margins to trade and transport industries, respectively. This was done with the help and technical support of IEA.

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