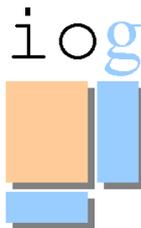# THE PROBLEM OF NEGATIVES IN INPUT-OUTPUT ANALYSIS: A REVIEW OF THE SOLUTIONS.

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The main models to construct technical coefficients are the industry technology model and the commodity technology model. The former yields nonnegative coefficients and the latter fulfills nice theoretical properties, such as price invariance. Although the models are very different, this paper presents a flexible formula that encompasses both of them. Various solutions to the problem of negatives of the commodity technology model—including replacements by industry technology based coefficients—are reviewed in our framework.

THE PROBLEM OF NEGATIVES IN INPUT-OUTPUT ANALYSIS: A REVIEW OF THE SOLUTIONS

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ABSTRACT: The main models to construct technical coefficients are the industry technology model and the commodity technology model. The former yields nonnegative coefficients and the latter fulfills nice theoretical properties, such as price invariance. Although the models are very different, this paper presents a flexible formula that encompasses both of them. Various solutions to the problem of negatives of the commodity technology model—including replacements by industry technology based coefficients—are reviewed in our framework.

KEYWORDS: Input-output analysis; technical coefficients; use and make matrices.
JEL CODES: C67, D57.

1. INTRODUCTION

The theoretically superior method for the construction of technical coefficients, the so-called commodity technology model, is plagued by the problem that it generates negative results. This explains the continuing popularity of the main competing method, the industry technology model, also as a basis for modifications of the commodity technology model. In this paper we present a formula that encompasses both models. It enables us to review the sources of negative coefficients and the procedures that have been proposed to remove them.

It is a bit surprising that an encompassing formula exists. After all, the industry and commodity technology models are very different. The industry technology model takes weighted averages of industry coefficients, where the latter are defined by dividing inputs by total industry output. The weights are market shares and the consequent commodity input coefficients are simple and nonnegative. The commodity technology model, however, involves an inversion of the output matrix. Commodity input coefficients are postulated and for each industry the implied input demand vector is equated with the observed use vector; a system of equations must be solved.

Following the United Nations System of National Accounts (1968) consider a *use* matrix $U = (u_{ij})_{i, j = 1, ..., n}$ comprising commodities i consumed by sectors j, and a *make* matrix $V = (v_{ij})_{i, j = 1, ..., n}$ showing the produce of sectors i in terms of commodities j. Notice that we consider square tables, with the same number of commodities as of industries. The issue is the derivation of input coefficients (a_{ij}). In other words, how can we estimate the amount of commodity i used for the production of one unit of commodity j?

Our point of departure is the amount of commodity i used by industry j (u_{ij}). The basic idea, common to the main methods of construction, is to subtract from u_{ij} the consumptions of commodity i used by industry j for its secondary products, $k \neq j$. The problem is that secondary outputs of industry j do not necessarily have the same input structure as in the industries where they are primary output. The flipside of the coin is that to compute the average input requirements of commodity j, we must *add* the secondary output to v_{jj} and the associated inputs to u_{ij}. The total input requirements of commodity i for making commodity j (as a single product) are thus:

$$u_{ij} - \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ijk} v_{jk} + \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ikj} v_{kj} ,$$

where a_{ijk} is the amount of commodity i used by industry j for making a unit of commodity k. Dividing by the total output of commodity j we obtain our general formula for technical coefficients:

$$a_{ij} = \frac{u_{ij} - \sum\limits_{\substack{k=1 \\ k \neq j}}^{n} a_{ijk} v_{jk} + \sum\limits_{\substack{k=1 \\ k \neq j}}^{n} a_{ikj} v_{kj}}{\sum\limits_{k=1}^{n} v_{kj}} . \tag{1}$$

We shall now show how this formula encompasses the main constructs of the literature, namely the commodity technology model and the industry technology model.

The **commodity technology model** assumes that all commodities have the same inputs structure irrespective the industry of fabrication:

$$a_{ijk} = a_{ik}, \quad \forall j = 1, 2, ..., n .$$

Therefore, (1) becomes:

$$a_{ij} = \frac{u_{ij} - \sum\limits_{\substack{k=1 \\ k \neq j}}^{n} a_{ik} v_{jk} + \sum\limits_{\substack{k=1 \\ k \neq j}}^{n} a_{ij} v_{kj}}{\sum\limits_{k=1}^{n} v_{kj}} , \tag{2}$$

which, in matrix terms, is:

$$A = \left(U - A \tilde{V}^{\mathrm{T}} + A \left(\widehat{\tilde{V}^{\mathrm{T}} e} \right) \right) \left(\widehat{V^{\mathrm{T}} e} \right)^{-1} =$$
$$= \left(U - A \tilde{V}^{\mathrm{T}} + A \left(\widehat{\tilde{V}^{\mathrm{T}} e} \right) \right) \left(\widehat{Ve} + \left(\widehat{\tilde{V}^{\mathrm{T}} e} \right) - \widehat{\tilde{V} e} \right)^{-1} , \tag{3}$$

and operating with (3), it yields,

$$U - A \tilde{V}^{\mathrm{T}} + A \left(\widehat{\tilde{V}^{\mathrm{T}} e} \right) = A \left(\widehat{V^{\mathrm{T}} e} \right) =$$
$$= A \left(\widehat{Ve} + \left(\widehat{\tilde{V}^{\mathrm{T}} e} \right) - \widehat{\tilde{V} e} \right) = A \widehat{Ve} + A \left(\widehat{\tilde{V}^{\mathrm{T}} e} \right) - A \left(\widehat{\tilde{V} e} \right),$$

which is the same as:

$$U - A\tilde{V}^{\mathrm{T}} = A\widehat{Ve} - A\left(\widehat{\widehat{V}e}\right) = A\left(\widehat{Ve} - \widehat{\widehat{V}e}\right) = A\left(\widehat{Ve - \tilde{V}e}\right) = A\left(\widehat{\widehat{V}e}\right) = A\hat{V}^{\mathrm{T}}$$

and:

$$U = A\tilde{V}^{\mathrm{T}} + A\hat{V}^{\mathrm{T}} = A\left(\tilde{V}^{\mathrm{T}} + \hat{V}^{\mathrm{T}}\right) = AV^{\mathrm{T}}.$$

Consequently, $A = UV^{\mathrm{T}}$. Notice that technical coefficients can be negative when the total consumption of input i for the making of secondary outputs of industry j, according to each one of these commodity technologies, is larger than the total use of commodity i by the industry j, either for its primary or secondary products.

The **industry technology model** assumes that all industries have the same inputs structure irrespective of the commodities they produce. This means that:

$$a_{ijk} = a_{ij}, \quad \forall k = 1, 2, ..., n.$$

Hence, (1) becomes into:

$$a_{ij} = \frac{u_{ij} - \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij} v_{jk} + \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ik} v_{kj}}{\sum_{k=1}^{n} v_{kj}},$$

which, in matrix terms, is:

$$A = \left(U - A\left(\widehat{\widehat{V}e}\right) + A\tilde{V}\right)\left(\widehat{V^{\mathrm{T}}e}\right)^{-1}, \tag{4}$$

and operating (4) properly,

$$U = A\left(\widehat{V^{\mathrm{T}}e}\right) + A\left(\widehat{\widehat{V}e}\right) - A\tilde{V} = A\left(\widehat{V^{\mathrm{T}}e} + \widehat{\widehat{V}e} - \tilde{V}\right),$$

and:

$$A = U\left(\widehat{V^{\mathrm{T}}e} + \widehat{\widehat{V}e} - \tilde{V}\right)^{-1} = U\left(\widehat{V^{\mathrm{T}}e} + \widehat{V}e - V\right)^{-1},$$

since $\tilde{V}e = Ve - \hat{V}e$ and $\tilde{V} = V - \hat{V}$.

Under the industry technology assumption no negative technical coefficient can be obtained in (4).

The commodity technology assumption is considered theoretically superior (Kop Jansen and ten Raa, 1990; Steenge, 1990; Konijn, 1994; Rainer, 1989; Mattey and ten Raa, 1997; ten Raa and Rueda Cantuche, 2003), but agreement is not uniform. For

instance, Mesnard (2002) stated that the commodity technology hypothesis has to be rejected since it breaks the linkages of commodity flows internal to the industries. No information is available to determine which industry supplies a positive variation in final demand of a particular commodity. Mesnard suggests that the industry technology model may be preferred even though it violates some axioms of Kop Jansen and ten Raa (1990).

2. SOURCES OF NEGATIVE COEFFICIENTS

Commodity technology model based coefficients may be negative for a number of reasons.

First, some commodities are produced with different technologies (Armstrong, 1975; ten Raa, Chakraborty and Small, 1984; ten Raa and van der Ploeg, 1989; Rainer and Richter, 1992; Konijn, 1994; Mattey and ten Raa, 1997). In this case, the commodity technology assumption is not valid.

Second, production classifications are heterogeneous (ten Raa, Chakraborty and Small, 1984; Rainer, 1989; Konijn, 1991; Rainer and Richter, 1992; Konijn, 1994; Konijn and Steenge, 1995; Mattey and ten Raa, 1997; Avonds and Gilot, 2002).

And third, the use and make data have errors of measurement (Armstrong, 1975; ten Raa, 1988; ten Raa and van der Ploeg, 1989; Rainer, 1989; Steenge, 1990; Mattey, 1993; Konijn, 1994; Konijn and Steenge, 1995; Avonds and Gilot, 2002).

In spite of these complications, the new handbook of the compilation of input-output tables (UN, 1993) and, even more recently, the European System of Accounts 1995 Input-Output Manual (EUROSTAT, 2002) recommend the use of the commodity technology assumption. Hence we must review procedures that have been suggested to remove the negative coefficients.

3. REMOVAL PROCEDURES

Since the pioneering work of Edmonston (1952, p. 569) several procedures remove the negatives that come with the commodity technology model.

3.1 Armstrong procedure (Armstrong, 1975)

Armstrong (1975) applied a hybrid technology assumption based on Gigantes (1970). Secondary products that yield negatives in the input-output table were subjected to the industry technology assumption. Hybrid methods require that the make matrix is split into two matrices, V_1 and V_2, where in this case the first one includes outputs for which the commodity technology assumption is made and the second includes those, which are to be treated on an industry technology assumption.

Particularly, in the United Kingdom (UK) Input-Output Tables for 1963, 55% of secondary production was treated on the industry technology assumption and the remaining 45% on the commodity technology hypothesis. However, in some cases this procedure leads to unacceptable situations. For instance, in Armstrong (1975) other food industries have as secondary outputs distribution services, which have large inputs of printing, while other food industries do not use much of them. Application of the industry technology assumption would result in a large amount of agricultural inputs into distribution services. In such cases disaggregation was adopted.

Formally, industry outputs of commodities for which the commodity technology assumption is made, are proportional to the output of each industry:

$$V_1^{\mathrm{T}} = C_1 \left(\widehat{V_1 e} \right).$$

and the market shares are denoted by:

$$V_2 = D_2^* \left(\widehat{V_2^{\mathrm{T}} e} \right).$$

Armstrong (1975) obtains two variants of technical coefficients matrices, depending on which of the following two assumptions is made. Industry outputs for which an industry technology assumption is made are proportional to the economy-wide outputs of each commodity whatever industry produces them. Commodity outputs for which an industry technology assumption is made are proportional to the economy-wide outputs of the producing industries (Armstrong 1975, pp. 74-76). Mathematically, the former assumption is denoted as:

$$V_2 e = D_2 \left(V^{\mathrm{T}} e \right),$$

whereas the latter is defined by:

$$V_2^{\mathrm{T}} e = C_2 \left(V e \right).$$

In the UK Input-Output Tables for 1963, Armstrong (1975) used the first assumption and the resulting technical coefficients matrix was given by:

$$A_H(U,V) = U\left(\widehat{Ve}\right)^{-1}\left(\left(\widehat{Ve_1}\right)V_1^{-\mathrm{T}}\left(I - \left(\widehat{V^{\mathrm{T}}e}\right)^{-1}\left(\widehat{V_2^{\mathrm{T}}e}\right)\right) + V_2\left(\widehat{V^{\mathrm{T}}e}\right)^{-1}\right).$$

Nevertheless, there were some remaining negatives that Armstrong (1975) set manually to zero or to a slightly positive value adjusting other entries of the input-output table so that overall accounting constraints in rows and columns still remain, as one expects when the commodity technology assumption continues to be used for part of the economy.

3.2 Almon procedure (Almon, 1970)

Almon (2000)[1] devised an iterative method that calculates non-negative technical coefficients matrices through a commodity-technology based algorithm. It is solved by the Seidel iterative process:

$$a_{ij}^{(h+1)} \sum_{k=1}^{n} v_{kj} = u_{ij} - \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ik}^{(h)} v_{jk} + \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj}. \tag{5}$$

Convergence is guaranteed if more than half of the production of a product is in its primary industry. Almon (2000) interprets this equation for the use of chocolate (i), in the production of cheese (j). Then, "the first term on the right of (5) tells us to begin with the chocolate purchases by the establishments in the cheese industry. The second term directs us to remove the amounts of chocolate needed for making the secondary products of those establishments by using our present estimate of the technology used for making those products, $a_{ik}^{(h)}$. Finally, the last term causes us to add back the chocolate used in making cheese in other industries." However, Almon's claim that the third term is exactly equal to the amount stolen, via second terms, from other industries, is not true. Unless all products are considered and not only cheese, the sum of the two latter terms of the right-hand side of (5) do not necessarily be null. Therefore, only by summing both sides of (5) over j, we will obtain the amount of input i used for the making of all products of the economy and not only for cheese (product j). That is,

$$\sum_{j=1}^{n}\left(a_{ij}^{(h+1)}\sum_{k=1}^{n}v_{kj}\right)=\sum_{j=1}^{n}u_{ij}-\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk}+\sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n}a_{ij}^{(h)}v_{kj}\,, \tag{6}$$

It is straightforward that the second and the third term of the right-hand side of (6) are indeed exactly the same with opposite signs. This does not mean of course that for each product j, both terms must coincide.

Proposition 1.

If V is non-singular and non-negative, the technical coefficients matrix A derived from $U = AV^{\mathrm{T}}$ will yield negative values if and only if for some product j,

$$u_{ij}<\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk}-\sum_{\substack{k=1\\k\neq j}}^{n}a_{ij}^{(h)}v_{kj}\,, \tag{7}$$

In short, when the difference between the amount of chocolate needed for making the secondary products of the cheese industry (j) and the amount of chocolate used by other industries to produce cheese is larger than the total purchases of chocolate by the establishments of the cheese industry (for making either primary or secondary products), negatives arise. The sufficiency proof follows from (5). For the necessity proof, let us assume a given negative technical coefficient, $a_{ij}^{(h+1)}<0$, since $\sum_{k=1}^{n}v_{kj}>0$, then $a_{ij}^{(h+1)}\sum_{k=1}^{n}v_{kj}<0$, and bearing in mind (5), it follows that:

$$a_{ij}^{(h+1)}\sum_{k=1}^{n}v_{kj}=u_{ij}-\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk}+\sum_{\substack{k=1\\k\neq j}}^{n}a_{ij}^{(h)}v_{kj}<0$$

and, consequently, $u_{ij}<\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk}-\sum_{\substack{k=1\\k\neq j}}^{n}a_{ij}^{(h)}v_{kj}.$

Proposition 2.

A technical coefficients matrix A calculated on the basis of $U = AV^{\mathrm{T}}$ will not yield negative values when for a non-singular and non-negative make matrix V,

$$\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk}<\sum_{\substack{k=1\\k\neq j}}^{n}a_{ij}^{(h)}v_{kj},\forall j\,. \tag{8}$$

---

[1] Almon first devised this procedure in Almon (1970).

Since $u_{ij} \geq 0$, and assuming $\sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ik}^{(h)} v_{jk} < \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj}$ or $\sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ik}^{(h)} v_{jk} - \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj} < 0$,

then it always occur that $u_{ij} - \left(\sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ik}^{(h)} v_{jk} - \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj} \right) > 0$, which, according to

Proposition 1, implies a non-negative value of *A*. The necessity proof does not hold

because $u_{ij} - \left(\sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ik}^{(h)} v_{jk} - \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj} \right) > 0$ does not guarantee that (8) will be fulfilled.

As it has been developed so far, negatives still can remain after having solved Almon's algorithm. In fact, the procedure to keep the negatives out of *A* consists of scaling down all components of the removal term, i.e. the second on the right-hand side of (5), to leave a zero balance. Then, the "total-stolen-from-other-industries" term is added gradually and not all at once. This process can be expressed in equations introducing in (5) scale factors, $s_{ij}^{(h)}$, which are defined by:

$$
s_{ij}^{(h)} = \begin{cases} \dfrac{u_{ij}}{\sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj}} & if \quad u_{ij} < \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj} \\[4mm] 1 & if \quad u_{ij} \geq \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj} \end{cases} . \tag{9}
$$

Equation (5) is then replaced by:

$$
a_{ij}^{(h+1)} \sum_{k=1}^{n} v_{kj} = u_{ij} - s_{ij}^{(h)} \sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ik}^{(h)} v_{jk} + \sum_{\substack{k=1 \\ k \neq j}}^{n} s_{ik}^{(h)} a_{ij}^{(h)} v_{kj} . \tag{10}
$$

However, as described in Almon (2000), only when the removal term is larger than the entry in the use matrix from which it is being removed - that is, when u_{ij} is lower than the negative term on the right-hand side of (5) -, scaling down is applied to leave a zero balance. Therefore, scale factors should be redefined replacing in $s_{ij}^{(h)}$,

$\sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ij}^{(h)} v_{kj}$ by $\sum_{\substack{k=1 \\ k \neq j}}^{n} a_{ik}^{(h)} v_{jk}$.

Though by summing both sides of (10) over j it is easy to see that the result is the same no matter what scale factor is defined, economic interpretation of the two latter terms of (10) is guaranteed in case we assume the redefined scale factor. Then, substituting in (10)[2], we obtain:

$$a_{ij}^{(h+1)}\sum_{k=1}^{n}v_{kj} = u_{ij} - \left(\frac{u_{ij}}{\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk}}\right)\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk} + \sum_{\substack{k=1\\k\neq j}}^{n}\left(\frac{u_{ik}a_{ij}^{(h)}v_{kj}}{\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk}}\right), \tag{11}$$

where it is straightforward that a zero balance is obtained by summing the two former terms of the right-hand side of (11). Finally, (11) results in:

$$a_{ij}^{(h+1)}\sum_{k=1}^{n}v_{kj} = \sum_{\substack{k=1\\k\neq j}}^{n}\left(u_{ik}\frac{a_{ij}^{(h)}v_{kj}}{\sum_{\substack{p=1\\p\neq k}}^{n}a_{ip}^{(h)}v_{kp}}\right), \tag{12}$$

where $a_{ij}v_{kj}$ represents the total inputs of commodity i used for making the secondary product j of industry k, and $\sum_{\substack{p=1\\p\neq k}}^{n}a_{ip}v_{kp}$, the total inputs of commodity i used for making all secondary products of industry k. Actually, in (12) the total requirements of inputs i needed by industry k (u_{ik}) is multiplied by the proportion of inputs i used for making secondary outputs of commodity j by industry k over the total consumption of these inputs for making all secondary products by the same industry. The result is the so-called: "total-stolen-from-other-industries" term (Almon, 2000). Finally, by summing both sides of (12) over j, it yields:

$$\sum_{j=1}^{n}\left(a_{ij}^{(h+1)}\sum_{k=1}^{n}v_{kj}\right) = \sum_{j=1}^{n}\sum_{\substack{k=1\\k\neq j}}^{n}\left(u_{ik}\frac{a_{ij}^{(h)}v_{kj}}{\sum_{\substack{k=1\\k\neq j}}^{n}a_{ik}^{(h)}v_{jk}}\right) = \sum_{j=1}^{n}u_{ij}\left(\frac{\sum_{\substack{j=1\\j\neq k}}^{n}a_{ij}^{(h)}v_{kj}}{\sum_{\substack{p=1\\p\neq k}}^{n}a_{ip}^{(h)}v_{kp}}\right) = \sum_{j=1}^{n}u_{ij}.$$

---

[2] Notice that for scale factor equals to one, (10) becomes (5) and it is easy to see that, by summing both sides of (5) over j, the result is given by the total uses of input i by industry j.

As a final result of the iterative process, by the choice of the scalar factors *s*, it is guaranteed that no negative technical coefficients will be obtained. In fact, this procedure actually converges but it is not clear to what. Ten Raa, Chakraborty and Small (1984) criticize this arithmetic manipulation arguing that it goes without justification, is arbitrary and depends on the choice of *V*-decomposition as well as the iteration scheme. Notwithstanding these arguments, Almon (2000) considers this procedure to be with perfectly reasonable economic interpretation and continues to use it during the last thirty years in the INFORUM model (Almon, 1991, 2000). Almon's approach is generally used when negatives arise because of the inexactness in make and use tables or because of slight differences in the technologies in different industries. More recently, the Belgian input-output tables (Avonds and Gilot, 2002) comprise a commodity-by-commodity input-output table using Almon's procedure to remove the minor negatives that could not be removed manually.

3.3 Rainer procedure (Rainer, 1989)

Rainer focuses on the fact that different data setups must be constructed since the purpose of making a homogeneous input-output table differs from the objective of making a descriptive make-use system in terms of National Accounts. Rainer (1989) and Rainer and Richter (1992) show for the 1976 Austrian use and make tables that when the commodity technology assumption is applied, some negatives can be removed with data rearrangements but not all of them. The remaining negatives were treated with the industry technology assumption. Basically, in most cases, these data rearrangements consist of giving answer to vertically integrated processes, features underlying classification systems, product mix problems and others mainly specific of the Austrian economy. In conclusion, data arrangement can be an answer to the problem of negatives when the commodity technology hypothesis is assumed.

3.4 Activity technology model (Konijn, 1994)

Konijn (1994) assumes that industries can produce commodities according to several production processes and that the same production process can be used by other industries. Moreover, production processes may have multiple outputs, as in the by-

product technology model described in Kop Jansen and ten Raa (1990). However, the resulting activity-by-activity input-output table is still not free of negatives. Konijn (1994) and Konijn and Steenge (1995) argue that remaining negatives clearly indicate that some classification adjustments must be made or some further research on errors data must be developed. Although the need of further information on the use and make system is required to apply the activity technology model, Statistics Netherlands actually adopted this way of removing negatives. In conclusion, Konijn (1994) proposes that we explicitly look at production processes instead of commodities and that we take the commodity classification of use and make matrices as an instrument rather than a fixed datum.

3.5 Steenge procedure (Steenge, 1990)

Steenge (1990) locates errors in such a way that minimal changes are needed to obtain a non-negative technical coefficients matrix. Unlike ten Raa (1988) and ten Raa and van der Ploeg (1989)—who use *a priori* estimated errors, Steenge proposes a way to locate errors *a posteriori*.

The United States available information in Steenge (1990) was performed with only 14 industries and commodities for 1977 and illustrates how very few corrections are required to obtain make and use matrices such that the commodity technology produces no negative coefficients. However, the adjustments lack statistical significance.

3.6 Stahmer procedure (Stahmer, 1985)

Stahmer (1985) established a modified transformation procedure based on the commodity technology model using special transformation matrices for certain rows and columns and using additional data. The procedure firstly defines a "special transformation table" for a particular input *k* as shown in Table 1.

According to the commodity technology assumption, technical coefficients for making the same product whatever industry produces them are coincident. Therefore, the output totals in each row are multiplied by the same input coefficient. In matrix terms, Stahmer (1985) define a "special transformation matrix" for each input *k* as

shown in Table 1: $S_k = \hat{a}_k V^T$ where $\hat{a}_k$ represents a diagonal matrix with the elements of the k-th row of the technical coefficients matrix in the diagonal. The column totals of each S_k show the use data available before the adjusting procedure and the row totals, the result of the transition process.

	Table 1. Stahmer procedure (1985)					
	Industries					**Totals**
	$a_{k1}v_{11}$	...	$a_{k1}v_{j1}$	...	$a_{k1}v_{n1}$	$a_{k1}\sum_{j=1}^{n} v_{j1}$
Commodities	**...**	...	...	...	...	...
	$a_{ki}v_{1i}$	...	$a_{ki}v_{ji}$	...	$a_{ki}v_{ni}$	$a_{ki}\sum_{j=1}^{n} v_{ji}$
	...	...	...	...	...	...
	$a_{kn}v_{1n}$	...	$a_{kn}v_{jn}$	...	$a_{kn}v_{nn}$	$a_{kn}\sum_{j=1}^{n} v_{jn}$
Totals	u_{k1}	...	u_{kj}	...	u_{kn}	

Source: Stahmer (1985).

Stahmer (1985)[3] proposes to use the special transformation matrix S_k in cases where the commodity technology model generates negatives. With additional available information, S_k is corrected in such a way that total uses of input k by each industry remain unchanged. However, since during the correction process available data about use or inputs requirements remain unchanged for every industry and commodity input, it may occur that several different input structures for one commodity can be established in certain rows and columns of A, which in fact introduce some distortion in terms of the commodity technology assumption (Konijn, 1994). Actually, the commodity technology hypothesis would not hold any more.

The German Federal Statistical Office applied this procedure for its 1980 input-output tables.

3.7 United States procedure (Young, 1986)

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[3] Stahmer (1982) originally includes also the value-added row into the use matrix.

The Bureau of Economic Analysis hybrid procedure (Guo, Lawson and Planting, 2002) first applies the commodity technology assumption to those secondary products that are not suitable for the industry technology model. All the resulting transfers of associated inputs and outputs are made at the same time as the use and make matrices are compiled. Hence, the published use and make tables actually includes all these transformations. However, some additional information on inputs structures is also used so that the commodity technology model cannot be applied in a fully mechanical way (redefinition process). The remaining secondary products are then transferred according to the industry technology assumption.

4. NON-NEGATIVITY CONDITIONS AND RECTANGULAR MATRICES

Konijn (1994) and Steenge (1990)[4] gave a mathematical answer to the question of when we can have a non-negative matrix A which satisfies the commodity technology assumption, for given rectangular use and make matrices. That is, a non-negative matrix A exists, satisfying $U = AV^T$, if and only if for any vector $s \geq 0$ with $V^T s > 0$, we have $Us \geq 0$ (*Mangasarian's theorem*). If s is interpreted as a vector of changes in activity levels this theorem states that if for some change in industry activity the output of all commodities is increased, then the use of none of the commodities is decreased. Otherwise, a firm could produce more of everything, while using less of something, which is strange according to Konijn (1994). Steenge (1990) argues that the resulting technical coefficients matrix only would represent imputed commodity specific input proportions, consistent with the overall use and make tables. Further research by industrial experts on empirical data is required to see if columns of the A matrix can be interpreted as production functions.

5. CONCLUSIONS: A GUIDING PROCEDURE

Once we have formalized the problem of how to construct a technical coefficients matrix and once we have discussed the different problems and solutions given to the drawbacks underlying the commodity technology model when the construction of a

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[4] For a full description of all theorems and propositions see Steenge (1990, p. 380) and Konijn (1994, pp. 139-142).

technical coefficients matrix is the main purpose, we conclude providing a guiding procedure.

Three reasons support the use of the commodity technology model. First, at a micro level data the commodity technology model seems to be a useful working hypothesis according to the test provided in Mattey and ten Raa (1997). Hence, disaggregation is commendable. Second, as shown in Steenge (1990) and Konijn (1994), it is theoretically possible to find a non-negative technical coefficients matrix consistent with the make-use system (even with rectangular use and make matrices), provided some adjustments are made. And third, the commodity technology assumption fulfills the axioms of Kop Jansen and ten Raa (1990).

Therefore, it seems that a commonly and desirable guide to proceed in order to construct technical coefficients matrices would consist in applying the commodity technology assumption, using afterwards location methods for errors of measurement (although these adjustments may be rejected statistically, they are supposed to be accepted by industrial experts) or negatives as indicators of errors of measurement and/or aggregation problems. Once these problems are solved no more negatives should arise. It is desirable to use the industry technology assumption just in the needed cases.

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