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Structural decomposition analysis (SDA) has been widely used to assess the relative importance of effects that together constitute a change in the variable of interest. A well known problem of SDA is that the results often depend strongly on the specific decomposition formula chosen, whereas numerous formulae are equivalent from a theoretical point of view. This non-uniqueness problem is often solved rather pragmatically, by reporting an average of (a subset of) all possible formulae. In previous works the Path Based SDA methodology has been proposed as an alternative approach to these average solutions. This technique incorporated some additional information of the factors to choose a specific decomposition formula. In this paper, we suggest that additional information of the variable of interest can be used with this same purpose too. We illustrate the method empirically by investigating the sources of growth in sectoral labor use in Spain, 1980-1994.

Path Based SDA with additional information of the dependent variable

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ABSTRACT

Structural decomposition analysis (SDA) has been widely used to assess the relative importance of effects that together constitute a change in the variable of interest. A well known problem of SDA is that the results often depend strongly on the specific decomposition formula chosen, whereas numerous formulae are equivalent from a theoretical point of view. This non-uniqueness problem is often solved rather pragmatically, by reporting an average of (a subset of) all possible formulae. In previous works the Path Based SDA methodology has been proposed as an alternative approach to these average solutions. This technique incorporated some additional information of the factors to choose a specific decomposition formula. This paper suggests that additional information of the variable of interest can be used with this same purpose too. We illustrate the method empirically by investigating the sources of growth in sectoral labor levels in Spain, 1986-1994.

Keywords: Structural decomposition analysis, maximum entropy, labor, Spain.

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1. Introduction

Among all the techniques related to Input-output (IO) framework, structural decomposition analysis (SDA) is a somewhat new methodology that has gained increasing popularity recently, principally since the eighties. An extensive overview of the methodology and its relatively early applications was provided by Rose & Casler (1996). Some recent applications of different sorts include De Haan (2001), Hoekstra & Van den Bergh (2003) and Dietzenbacher *et al.* (2000, 2004). Dietzenbacher & Los (1998, 2000) showed that SDA results should be taken with care, because several methodological problems affect to the techniques employed mainly related to the non-uniqueness in its solutions, which is not only a theoretical issue: relative contributions of distinct sources of change depend considerably on the specific decomposition form chose.

Dietzenbacher & Los (1998) showed that the number of theoretically equivalent forms amounts to $n!$, in which n represents the number of distinct sources of change. Their admittedly pragmatic solution is to present averages of results obtained for all decomposition forms or for a well-defined small subset of forms. In Fernandez´s Ph.D thesis (2004) the Path Based method was proposed: this work argued that some available additional information could be used to divide the interaction terms in a way that fits the data better than implied by simply taking averages. The additional data were used in a Maximum Entropy (ME) estimation procedure to arrive at parameter estimates that determine the temporal paths followed by the factors. These estimates specify a unique division of the interaction terms.[1] Basically, the additional information considered in this approach were data of some of the factors involved in the decomposition problem for periods inbetween the initial and final time period. In this paper we extend this approach, considering now the possibility that the only available information concerns the dependent variable whose temporal change we want to decompose.

The paper is organized as follows. In Section 2, we briefly present the "non-uniqueness" problem in SDA in formal terms by means of a simple decomposition analysis with two determinants; secondly, extending this analysis to the general case; and, finally, mentioning solutions proposed previously in the IO literature. Section 3 shows the basis of the so-called Path Based (PB) SDA, which offers a much broader class of solutions than those introduced in Section 2. One particular solution is given by a specific division of the interaction terms, and divisions are characterized by the parameters that appear in the temporal paths followed by the determinants. If these parameters can be estimated, we will obtain an unique solution to the decomposition, solving the non-uniqueness problem. In Section 4, the principles of ME estimation are highlighted, and we show how ME estimation techniques can be used to estimate the parameters of interest. Section 5 studies how additional information of the dependent variable can be used to implement the ME approach. In Section 6 we present an empirical illustration of the approach. We will study changes in labor requirements in Spanish sectors between 1986 and 1994. Our aim is to assess

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1 Maximum Entropy econometrics and strongly related Cross Entropy methods have been used in an intersectoral setting before. See, for example, Golan *et al.* (1994) and Robinson *et al.* (2001) for methods to estimate missing data in input-output tables and social accounting matrices.

the importance of sector-specific changes in the workforce use per unit of output on the one hand, and structural effects as a consequence of changes in the matrix of input coefficients and the vector of final demands on the other. We show that the contributions obtained often deviates substantially from the average solutions analyzed by Dietzenbacher & Los (1998), among others. Moreover, we also compare the outcomes obtained by different approaches with an annual average decomposition, in order to check which of them yields the closest solution. Section 7 presents the main conclusions of the paper.

2. SDA and the Non-Uniqueness Problem

From the most basic equation is **q**=**Lf** in IO analysis, where sectoral gross output levels **q** are expressed as the product of the Leontief inverse matrix **L** and the vector of sectoral final demand levels **f**, the objective of SDA is to measure the part of the variations in **q** that can be attributed to differences in **L** and the part caused by differences in **f**. This problem appears very often in other subdisciplins in economics[2], so we will explain the more traditional approaches and our new approach in terms of a more general notation.

The starting point is a dependent variable z defined as the product of a set of n factors (or, determinants)[3] x_1, x_2, ..., x_n. That is:

$$z = x_1 x_2 ... x_n \tag{1}$$

A fundamental assumption is that the factors can be assumed to be independent, not only in a mathematical sense (see Dietzenbacher and Los, 2000, for an account of problems related to mathematical dependency of determinants) but also from an economic-theoretical viewpoint. That is, each determinant could change without an necessarily accompanying change in the values of one or more of the other determinants. Without loss of generality, we will assume that the difference in z to be studied relates to a difference over time. Denoting the value of z in the initial period 0 by z^0 and its value in the final period 1 as z^1, we can write

$$z^0 = x_1^0 x_2^0 ... x_n^0 \tag{2}$$
$$z^1 = x_1^1 x_2^1 ... x_n^1 \tag{3}$$

To decompose the change in z, either additive or multiplicative approaches can be chosen. Although this last approach is gaining popularity in IO analysis since the paper by Dietzenbacher *et al.* (2000), we will focus on the probably more popular additive decomposition form, which is

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2 Actually, Fernández (2004, Chapter 4) offers an application of our methodology to a shift-share analysis of employment growth in Spanish regions.

3 The variables and the factors can be represented by scalars, vectors and/or matrices. Throughout the paper, we adopt the convention that scalars are represented by italic lowercase symbols, (column) vectors by lowercase bold symbols and matrices by bold capitals. Primes denote transposition and hats indicate diagonal matrices.

based on the *differences* between the left-hand sides and the right-hand sides of equations (3) and (2). We obtain:

$$\Delta z = z^1 - z^0 = x_1^1 x_2^1 \ldots x_n^1 - x_1^0 x_2^0 \ldots x_n^0 \tag{4}$$

The objective of additive decomposition analyses is now to express the value of the left-hand side as the sum of the respective effects of every determinant and to explain the nature of the non-uniqueness problem, we rely on the case in which n=2. For notational convenience, we will denote the factors by x and y. Hence, the temporal change in z is:

$$\Delta z = z^1 - z^0 = x^1 y^1 - x^0 y^0 \tag{5}$$

Now, by adding and subtracting $x^0 y^1$ in (5), we obtain:

$$\Delta z = x^1 y^1 - x^0 y^0 + x^0 y^1 - x^0 y^1 = \left(x^1 - x^0\right) y^1 + x^0 \left(y^1 - y^0\right) \tag{6}$$

and:

$$\Delta z = \Delta x y^1 + x^0 \Delta y \tag{7}$$

The first term on the right side of (7) represents the effect of changes in x to the actual change in z, and the second term quantifies the contribution of changes in variable y. The problem arises because different contributions could have been obtained if we had added and subtracted $x^1 y^0$ in (5) instead of $x^0 y^1$. In this case, we would have obtained:

$$\Delta z = \Delta x y^0 + x^1 \Delta y \tag{8}$$

The contributions of changes in x and y as obtained by expressions (7) and (8) can differ quite a bit and choosing one of them is an arbitrary decision.[4] As a pragmatic solution, authors have traditionally applied average solutions of expressions (7) and (8). As Dietzenbacher & Los (1998) pointed out, this is equal to using midpoint weights if and only if two determinants are discerned.

$$\Delta z = \Delta x y^{(1/2)} + x^{(1/2)} \Delta y \tag{9}$$

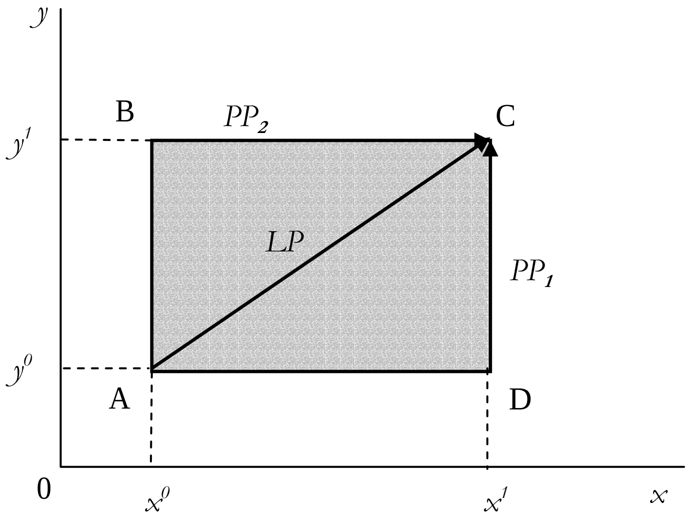
where,

[4] We only consider "exhaustive" decomposition forms, which implies that the full effect is attributed to changes in the exogenous determinants. An example of a "non-exhaustive" or "approximate" (Dietzenbacher & Los, 1998) decomposition form is $\Delta z = \Delta x y^0 + x^0 \Delta y + \Delta x \Delta y$. The last term is often labelled the "interaction effect". In some cases, approximate forms may be preferred over exhaustive forms, for example if a clear economic interpretation can be given to the interaction term. If n>2, however, approximate decompositions will contain a number of interaction terms, for which no straightforward interpretation is available. In such cases, we feel that exhaustive decomposition forms are most appropriate.

$$x^{(\frac{1}{2})} = \frac{x^0 + x^1}{2} \quad \text{and} \quad y^{(\frac{1}{2})} = \frac{y^0 + y^1}{2}$$

This discussion is graphically summarized by Figure 1, which was originally proposed by Sun (1998). The whole issue is about the treatment of the upper right rectangle (ABCD), the interaction effect. Equation (7) suggests to attribute it completely to the change in x, whereas equation (8) would attribute it completely to the change in y. Consequently, the contributions for x and y obtained by both expressions can imply remarkable differences, which depend on the size of the interaction term $\Delta x \Delta y$.

Figure 1. Polar and straight-line paths



The specification of a temporal path for the determinants implies a particular decomposition form to split-up the interaction term. We will get back to the issue of temporal paths in much more detail in the next section. For now, it should be noted that path PP_1 would mean that the effect of determinant x would be $\Delta x y^0$, and the effect of determinant y would be $x^1 \Delta y$. If we suppose that the temporal path between the initial and the final period is path PP_2, the respective contributions for determinants x and y would be $\Delta x y^1$ and $x^0 \Delta y$. Taking the average of these two alternative paths would imply an equal division of the interaction rectangle. It can easily be seen that taking the midpoint weights would yield an identical result. This result is also attained by Sun's (1998) method, which amounts to attribute halves of the interaction effect to the effects of changes in the two determinants. This amounts to drawing a straight line (LP) from (x^0, y^0) to (x^1, y^1).[5]

In the general case, in which z is the product of n determinants, the number of possible basic decompositions such as those corresponding to PP_1 and PP_2 is increased, now being equal to the

5 Sun's (1998) straight line can be considered as a special case of the continuous-time approach we will discuss below. Sun himself refers to his solution as the implication of a "jointly created and equally distributed" principle (Sun, 1998, p. 88).

number of possible permutations for n variables. Therefore, $n!$ forms could be obtained to decompose the change Δz. Specific cases among these are

$$\Delta z = \Delta x_1 x_2^0 \ldots x_n^0 + x_1^1 \Delta x_2 \ldots x_n^0 + \ldots + x_1^1 x_2^1 \ldots \Delta x_n \quad (10)$$

$$\Delta z = \Delta x_1 x_2^1 \ldots x_n^1 + x_1^0 \Delta x_2 \ldots x_n^1 + \ldots + x_1^0 x_2^0 \ldots \Delta x_n \quad (11)$$

These expressions are usually called "polar decompositions" (Dietzenbacher & Los, 1998), because the expressions for the effects are characterized by identical indexes for all determinants on both the left hand-side and right hand-side of the Δx_i factor.[6] The absence of uniqueness in the solutions leads to the arbitrary choice for one of the $n!$ possibilities, or alternatively one could obtain an average solution. As Dietzenbacher & Los (1998) showed, the average of the two polar decompositions is usually very close to the average taken over all $n!$ forms. They also show that a midpoint weighted formula is not exhaustive if $n>2$.

In the next sections, we will study the main features of a general method of decomposition that overcomes many of the limitations of the SDA approaches discussed so far. It allows us to obtain non-arbitrary solutions to measure the effects of the determinants of a change.

3. The Path Based Approach

In this section, a framework for an alternative decomposition method will be sketched. It builds on the earlier work by Vogt (1978), where the relevance of the temporal paths of the factors for the measurement of their contributions was pointed out. It is also connected with more recent works by Hoekstra & Van den Bergh (2002) and, in particular, Harrison *et al.* (2000), who introduced the basics of what we will call the Path Based (PB) approach. The alternative setup starts from the premise that both the value of z and the value of the determinants x_i have changed continuously over time, between time 0 and time 1. Hence, we can write:

$$z(t) = x_1(t) x_2(t) \ldots x_n(t) \quad (12)$$

and, assuming differentiability of each $x_i(t)$ an infinitesimal change in z can be expressed as

$$dz = \frac{\partial z}{\partial x_1}\frac{dx_1}{dt}dt + \ldots + \frac{\partial z}{\partial x_n}\frac{dx_n}{dt}dt \quad (13)$$

Finally, the total change in z can be expressed as the sum of all the infinitesimal changes between time 0 and time 1:

$$\Delta z = \int_{t=0}^{t=1} \frac{dz}{dt} dt = \int_{t=0}^{t=1} \sum_{i=1}^{n} \frac{\partial z}{\partial x_i}\frac{dx_i}{dt} dt \quad (14)$$

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6 In fact, this property is also fulfilled by the decomposition forms corresponding to PP_1 and PP_2. We will therefore denote such paths as "polar paths" (PP).

The effects of the determinants x_i can now be written as:

$$\Delta x_i \text{effect} = \int_{t=0}^{t=1} \frac{\partial z}{\partial x_i} \frac{dx_i}{dt} dt = \int_{t=0}^{t=1} \prod_{j \neq i}^{n} x_j \frac{dx_i}{dt} dt \qquad (15)$$

Equation (15) shows that the derivatives of the determinants x_i to time t play an important role in the size of the effects attributed to changes in these determinants. Consequently, the specification of the temporal path that each factor follows between initial and final periods, $x_i(t)=f_i(t)$, can have a big impact on the measurement of their effects that together add up to the variation in z. Harrison *et al.* (2000) proposed the solution arrived at by assuming straight-line paths of the variables x_i:

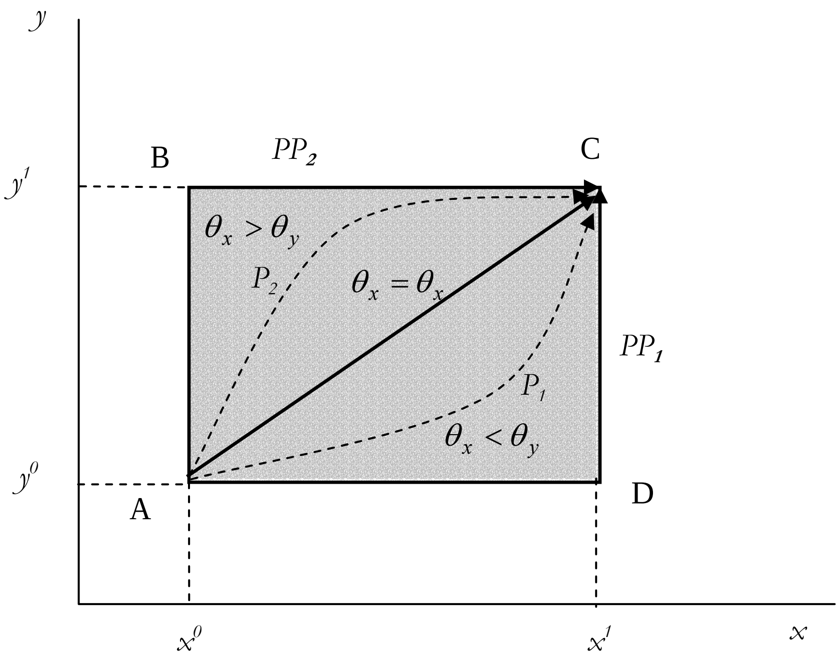
$$x_i(t) = x_i^0 + \left(x_i^1 - x_i^0\right)t = x_i^0 + \Delta x_i t \qquad (16)$$

Actually, this approach yields the same solution as Sun's (1998) 'equal shares' method. However, empirical values of the variables x and y at, for example, t=0.5 might be such that the straight line assumption is very unlikely to be tenable. In previous works, we suggested a method to take such information explicitly into account in attributing parts of the interaction effects to the effects of the respective determinants. The methodological innovation proposed was to relax the strict assumption of a straight line, by considering more flexible forms for the functions $f_i(t)$. In order to preserve possibilities to estimate the parameters that characterize the time-paths of the variables, we choose to consider a specific class of monotonic functions:

$$x_i(t) = x_i^0 + \Delta x_i t^{\theta_i}; \ \forall \, \theta_i > 0 \qquad (17)$$

The class of paths considered contains all possible monotonic paths for x_0 to x_1 that do not have inflexion points. This is a limitation for sure, because our class of paths do not cover those that contain values that are below the initial value or exceed the final value (assuming, without loss of generalization that x_1 is larger than x_0). Obviously, the temporal path of x_i will be a straight line if θ_i equals 1. If this holds for all i (i=1, ..., n), the solution obtained by the method introduced here will be identical to Harrison's et al. (2000) solution. By plotting a diagram for two determinants comparable to Figure 1, we can show that the class of time paths implied by the still relatively simple expression in equation (17) comprises a nicely defined set of time paths (see Figure 2).

Figure 2. Generalized monotonic temporal paths



The basic idea is that the specific path implied by the parameter values θ_i determines the shares of the interaction effect that is attributed to the distinct determinants. The "polar" paths PP_1 ($\theta_x / \theta_y \rightarrow 0$) and PP_2 ($\theta_x / \theta_y \rightarrow \infty$), and the straight-line path ($\theta_x / \theta_y = 1$) are included as special cases of this general class. P_1 and P_2 are intermediate cases. In a situation like the one represented by P_1, a larger part of the interaction effect is attributed to determinant y than if a situation better reflected by P_2 would occur.

For the most general case in which a change in z is decomposed into the effects of n determinants x_i (see equation (12)), the expression for the respective contributions for any possible set of n time paths was already given in equation (15). Substituting the more specific temporal paths assumed in equation (17) into equation (15), we can write

$$\Delta x_i \text{ Effect} = \int_{t=0}^{t=1} \prod_{j\neq i}^{n} x_j \frac{dx_i}{dt} dt = \left[\prod_{j<i}^{i-1} x_j^0\right] \Delta x_i \left[\prod_{j>i}^{n} x_j^0\right] + \tag{18a}$$

$$+ \sum_{j\neq i}^{n} \left[\frac{\theta_i}{\theta_i + \theta_j} \prod_{k<i}^{i-1} x_k^0 \Delta x_i \prod_{i<k<j}^{j-1} x_k^0 \Delta x_j \prod_{k>j}^{n} x_k^0 \right] + \tag{18b}$$

$$+ \sum_{j\neq i}^{n} \sum_{l\neq j,i}^{n} \left[\frac{\theta_i}{\theta_i + \theta_j + \theta_l} \prod_{k<i}^{i-1} x_k^0 \Delta x_i \prod_{i<k<j}^{j-1} x_k^0 \Delta x_j \prod_{j<k<l}^{l-1} x_k^0 \Delta x_l \prod_{k>l}^{n} x_k^0 \right] + \tag{18c}$$

. . .

$$+\frac{\theta_i}{\sum_{j=1}^{n}\theta_j}\left[\prod_{j=1}^{n}\Delta x_j\right] \tag{18d}$$

The first term in this sum shows the smallest contribution for determinant x_i, which is given by its growth Δx_i weighted by the initial values of the other variables.[7] It does not contain any part of the interaction effects. The remaining terms show a set of interaction effects between the growth of groups of determinants, also weighted by the initial values of the remaining determinants. The distribution of these joint effects clearly depends on the θ_i values. Multiple joint effects between the determinants exist. More specifically, there are $\binom{n-1}{1}$ possibilities of interaction between x_i and each one of the remaining $n-1$ determinants, $\binom{n-1}{2}$ terms measuring the joint effect of x_i with groups of $n-2$ determinants, etc. In general, in the expression for the effect of x_i there will be $\binom{n-1}{k}$ terms for the joint effects with groups of *k* determinants. The last terms (in equation 21d), shows the part of the joint contribution of all the determinants to the interaction effect attributed to x_i.

The importance of the values of the θ_i parameters for the measurement of the determinant's contributions is clear from equation (18). The higher the value of θ_i in comparison to the remaining θ_j, the greater the portions of the interaction effects attributed to x_i and, thus, the greater its contribution to the whole change in variable *z*. To illustrate this idea, it is helpful to give extreme values to a parameter θ_i. Let us suppose firstly that θ_i tends to its minimum value, *i.e.* we are supposing that it is very close to zero. In this case we obtain:

$$\lim_{\theta_i\to 0}\Delta x_i \text{ effect} = \left[\prod_{j<i}^{i-1} x_j^0\right]\Delta x_i\left[\prod_{j>i}^{n} x_j^0\right] = x_1^0 x_2^0 \ldots x_{i-1}^0 \Delta x_i x_{i+1}^0 \ldots x_n^0 \tag{19}$$

This would be the case when the effect of changes in variable x_i is at its smallest, because we are weighting it by the remaining determinants at their initial values. It should be noted that equation (19) is one of the *n!* feasible solutions obtained by SDA. The opposite situation will happen if we suppose that parameter θ_i has a much higher value than the rest of parameters θ_j. Then, the contribution of x_i will be:

$$\lim_{\theta_i\to\infty}\Delta x_i \text{ effect} = x_1^1 x_2^1 \ldots x_{\text{i-1}}^1 \Delta x_i x_{\text{i+1}}^1 \ldots x_{\text{n}}^1 \tag{20}$$

In such a case, the contribution of x_i to changes in variable *z* is as large as it can be, since we are weighting its variation by the remaining determinants measured at their final values. Between

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[7] For the sake of simplicity, let us hereafter suppose a situation in which $\Delta x_i \geq 0$; $i = 1,\ldots,n$.

these two extreme situations there exists a infinite range of possible contributions for each determinant, which depend on the value of parameters θ_i. All solutions obtained by SDA techniques are included in this range.

As we mentioned before, it is nowadays common practice in SDA analyses to present averages over decomposition forms. The average over all *n!* decomposition forms could be obtained by the PB approach as well. If we would not have any information on the evolution of the determinants over time other than the initial and the final observation, it would be most plausible to assume that the temporal path parameters are equal to each other ($\theta_1 = \theta_2 = \ldots = \theta_n$). According to equation (18) we would find

$$\Delta x_i \text{ effect} = \int_{t=0}^{t=1} \prod_{j \neq i}^{n} x_j \frac{dx_i}{dt} dt = \left[\prod_{j<i}^{i-1} x_j^0 \right] \Delta x_i \left[\prod_{j>i}^{n} x_j^0 \right] + \tag{21a}$$

$$+ \sum_{j \neq i}^{n} \left[\frac{1}{2} \prod_{k<i}^{i-1} x_k^0 \Delta x_i \prod_{i<k<j}^{j-1} x_k^0 \Delta x_j \prod_{k>j}^{n} x_k^0 \right] + \tag{21b}$$

$$+ \sum_{j \neq i}^{n} \sum_{l \neq j,i}^{n} \left[\frac{1}{3} \prod_{k<i}^{i-1} x_k^0 \Delta x_i \prod_{i<k<j}^{j-1} x_k^0 \Delta x_j \prod_{j<k<l}^{l-1} x_k^0 \Delta x_l \prod_{k>l}^{n} x_k^0 \right] + \tag{21c}$$

$$\cdot \quad \cdot \quad \cdot$$

$$+ \frac{1}{n} \left[\prod_{j=1}^{n} \Delta x_j \right] \tag{21d}$$

The interaction effects are thus shared proportionally to the changes in the values of the determinants. This is identical to the solution proposed by Sun (1998) discussed in the previous section. In spite of the similarity between the numerical outcomes for the mean of the two polar decompositions only and the mean of all *n!* decompositions (Dietzenbacher & Los, 1998), the mean of the polar decompositions cannot be obtained by means of specifying values for θ_i in the above-mentioned PB approach.[8] In the next sections we will turn to methods to infer on plausible values for θ_i, which allow us to apply equation (18) to interesting empirical problems.

4. Generalized Maximum Entropy Econometrics With Non-Linear Constraints

In the previous section, we found that taking the mean contributions of all decomposition forms is the most reasonable solution to the non-uniqueness problem if the researcher has no information at all about the time paths of the determinants. In many cases, however, more information than the values of the determinants at t=0 and t=1 is available. Anyway, estimation of the parameters θ_i is generally not possible by means of classical econometric estimation procedures like least squares estimation, since the amount of data is quite limited. In Fernández *et al.* (2005) such a situation is studied when the available information are the values of one or more

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[8] See Fernández (2004, pp. 36-39) for a proof.

of the determinants at intermediate points in time. The estimation procedure followed is based on maximum entropy (ME) econometrics, a collection of tools that can be very convenient to use scarce additional information in producing estimates for the temporal path parameters θ_i.[9]. In this section, we will explain how a generalization of ME (GME) can be used when the only available information comes as a set of non-linear constraints.

The starting point will be a random variable x which can get values $\{x_1,...,x_K\}$ with a distribution of probabilities $\mathbf{p} = p_1, p_2,..., p_K$ that is unknown for the researcher and has to be recovered. Following the formulation of Shannon (1948), the entropy of this distribution $\mathbf{p}$ will be

$$H(\mathbf{p}) = -\sum_{i=1}^{K} p_i \ln p_i \qquad (22)$$

The entropy measure H indicates the 'uncertainty' and reaches its maximum when $\mathbf{p}$ is a uniform distribution ($p_i = \frac{1}{K}, \forall \mathrm{i} = 1,..., \mathrm{K}$). If some information (*i.e.*, observations) is available, H can also be used to estimate $\mathbf{p}$. Suppose that there are T observations $\{y_1, y_2,..., y_T\}$ available such that

$$\sum_{i=1}^{K} p_i f_t(x_i) = y_t, \; 1 \leq t \leq T \qquad (23)$$

with $\{f_1(x), f_2(x),..., f_T(x)\}$ a set of known functions representing the relationships between the random variable x and the observed data $\{y_1, y_2,..., y_T\}$. In such a case, the ME principle can be applied to recover the unknown probabilities. This principle is based on the selection of the probability distribution that maximizes equation (22) among all the possible probability distributions that fulfill (23). The following constrained maximization problem is posed:

$$\underset{\mathbf{p}}{Max}\, H(\mathbf{p}) = -\sum_{i=1}^{K} p_i \ln p_i \qquad (24)$$

subject to:

$$\sum_{i=1}^{K} p_i f_t(x_i) = y_t, \; \forall t = 1,..., T$$

$$\sum_{i=1}^{K} p_i = 1$$

In this problem, the last restriction is just a normalization constraint that guarantees that the estimated probabilities sum to one, while the first T restrictions guarantee that the recovered distribution of probabilities is compatible with the data for all T observations. By solving this program, one "chooses" the estimates $\hat{\mathbf{p}}$ among all the distribution probabilities consistent with

9 See Kapur & Kesavan (1992) or Golan *et al.* (1996) for a detailed analysis of properties of the estimators obtained by means of these techniques.

the information available, being $\hat{\mathbf{p}}$ the one that maximizes the entropy. In other words, $\hat{\mathbf{p}}$ is the one that implies using only the amount of information involved in the T observations. It is important to note that even for T=1 (a situation with only one observation), the ME approach yields an estimate of the probabilities. Hence, in situations in which the number of observations is not large enough to apply econometrics based on limit theorems, this approach can be used to obtain robust estimates of unknown parameters.[10]

The above-sketched procedure has been generalized (GME) to the estimation of unknown parameters for traditional linear models (see Paris & Howitt, 1998; or Golan *et al.*, 2001 as examples), although some recent works have advocated the use of GME when the model to estimate is non-linear (see Grendar & Grendar, 2004). To illustrate this in a simple way, let us suppose that the model to estimate has the following form:

$$y_t = x_{1t}^{\theta_1} + x_{2t}^{\theta_1} + \ldots + x_{nt}^{\theta_n} + e_t \qquad (25a)$$

Or, for all the T observations:

$$\mathbf{y} = \mathbf{x}_\mathbf{1}^{(\theta_1)} + \mathbf{x}_\mathbf{2}^{(\theta_2)} + \ldots + \mathbf{x}_\mathbf{n}^{(\theta_n)} + \mathbf{e} \qquad (25b)$$

where $\mathbf{y}$ is a $(T \times 1)$ vector of observations for y, $\mathbf{x_i}$ is a $(T \times 1)$ vector of observations for the x_i variables ($\forall i = 1,\ldots,n$), θ_i is an unknown parameters to be estimated ($\forall i = 1,\ldots,n$), and $\mathbf{e}$ is a $(T \times 1)$ vector reflecting the random term of the linear model. For each θ_i, it will be assumed that there is some information about its $M \geq 2$ possible realizations by means of a 'support' vector $\mathbf{b}' = (b_1,\ldots,b^*,\ldots,b_M)$, the elements of which are symmetrically distanced around a central value $\theta_i = b^*$ (the prior expected value of the parameter), with corresponding probabilities $\mathbf{p}'_\mathbf{i} = (p_{i1},\ldots,p_{iM})$. For the sake of convenient exposition, it will be assumed that the M values are the same for every parameter, although this assumption can easily be relaxed. Consequently, Now, each parameter θ_i can be written as

$$\theta_i = \sum_{m=1}^{M} p_{im} b_m , \ \forall i = 1,\ldots,n \qquad (26)$$

For the random terms, a similar approach is chosen. To express the lack of information about the actual values contained in $\mathbf{e}$, we assume a distribution for each e_t, with a set of $H \geq 2$ values $\mathbf{v}' = (v_1,\ldots,v_H)$ with respective probabilities $\mathbf{w}'_\mathbf{t} = (w_{t1}, w_{t2},\ldots,w_{tH})$.[11] Hence, we can write

[10] Golan *et al*. (1996, p. 12) contains a simple, classic example of this technique, the so called "dice problem". A disadvantage of ME estimators is that comparisons of means and variances of estimators are not possible. Such comparisons are common practice in classical least squares and maximum likelihood econometrics.

[11] Usually, the distribution for the errors is assumed symmetric and centered about 0, therefore $v_1 = -v_J$.

$$\mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ \dots \\ e_T \end{bmatrix} = \mathbf{V}\mathbf{w} = \begin{bmatrix} \mathbf{v}' & \mathbf{0} & . & \mathbf{0} \\ \mathbf{0} & \mathbf{v}' & . & \mathbf{0} \\ . & . & . & . \\ \mathbf{0} & \mathbf{0} & . & \mathbf{v}' \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \dots \\ \mathbf{w}_T \end{bmatrix} \tag{27}$$

and the value of the random term for an observation t equals

$$e_t = \mathbf{v}'\mathbf{w}_\mathbf{t} = \sum_{h=1}^{H} v_h w_{th} \ ; \ \forall t = 1,...,T \tag{28}$$

And, consequently, equation (25b) can be transformed into

$$\mathbf{y} = \mathbf{x}_1^{\left(\sum_{m=1}^{M} b_m p_{1m} \right)} + \mathbf{x}_2^{\left(\sum_{m=1}^{M} b_m p_{2m} \right)} + ... + \mathbf{x}_\mathbf{n}^{\left(\sum_{m=1}^{M} b_m p_{nm} \right)} + \mathbf{vW} \tag{29}$$

and the ME program to estimate the $n+T$ probability distributions is the following:

$$\underset{\mathbf{p},\mathbf{w}}{Max}\, H(\mathbf{p},\mathbf{w}) = -\sum_{i=1}^{n}\sum_{m=1}^{M} p_{im}\, ln(p_{im}) - \sum_{t=1}^{T}\sum_{h=1}^{H} w_{th}\, ln(w_{th}) \tag{30}$$

subject to:

$$\sum_{m=1}^{M} p_{im} = 1, \ \forall i = 1,...,n$$

$$\sum_{h=1}^{H} w_{th} = 1, \ \forall t = 1,...,T$$

$$y_t = \sum_{i=1}^{n} \left[x_{it}^{\left(\sum_{m=1}^{M} b_m p_{im} \right)} \right] + \sum_{h=1}^{H} v_h w_{th} =, \ \forall t = 1,...,T$$

The estimated probabilities allow us to obtain estimations for the unknown parameters. The estimated value of θ_i will be[12,13]:

$$\hat{\theta}_i = \sum_{m=1}^{M} \hat{p}_{im} b_m \ , \ \forall i = 1,...,n \tag{31}$$

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12 The construction of the vector **b** is based on the researcher's prior knowledge (or beliefs) about the parameter. Golan *et al.* (1996, chapter 8) devote more attention to consequences of choices concerning the elements of the vector **b**. An almost universal result is that wider bounds can be used without substantial consequences for the characteristics of the estimators.

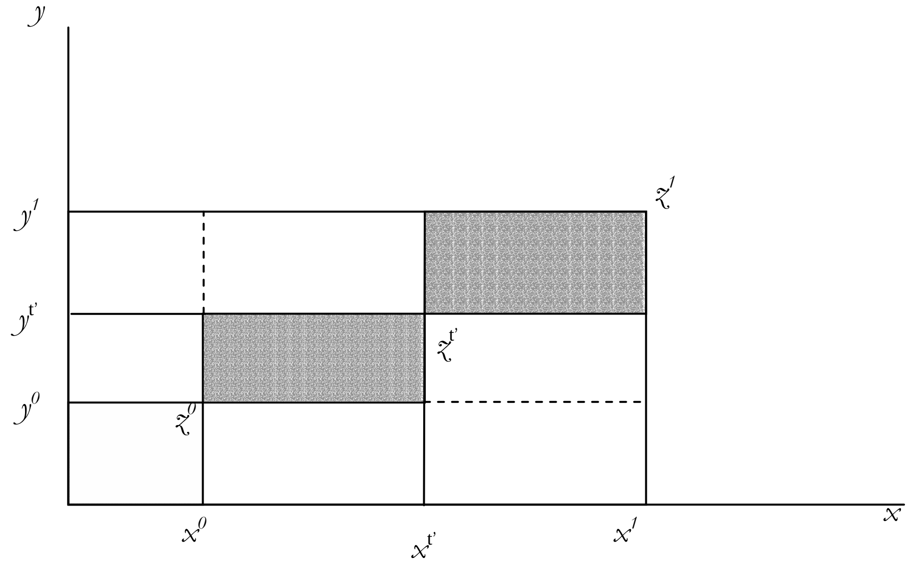
13 Fernández (2004, pp. 69) proves that the solution of the constrained maximization problem (36) without additional information yields estimates equal to the expected value b^* of the prior distribution.

This approach can be applied to the decomposition problem studied in the previous section, since limited additional information would enable us to obtain estimates of the parameters that determine the contribution of each determinant to the total change that has actually been observed. In other words, non-arbitrary solutions to the decomposition problem could be obtained. In the next section a situation with availability of additional data for $z(t)$ will be considered, as well as the way to estimate the effects of the factors to the total change Δz using this technique.

5. Incorporating Additional Information of *z(t)*

If there is some available additional information of intermediate periods between $t=0$ and $t=1$, this can help to reduce the non-uniqueness problem in SDA. Suppose that for the simplest case where $z(t)=x(t)y(t)$, we know the values of $x(t')$ and $y(t')$, being $0<t'<1$. In such a case, see that if we can incorporate the additional information and consider a two-stages decomposition $z^1-z^0=\left(z^1-z^{t'}\right)+\left(z^{t'}-z^0\right)$. Figure 3 illustrate this situation:

Figure 3. A two-stages decomposition



The sum of the interaction terms for the two stages of the decomposition (two grey shaded areas) is smaller than the interaction term $\Delta x \Delta y$ that appears in Figures 1 y 2. If the number of stages is increased, *i. e.* if the intermediate periods with observations for factors x and y increases, the size of the interaction terms decreases and this also reduces the seriousness of the non-uniqueness problem[14]. Unfortunately, such a solution is not always possible. In fact, the unavailability of regular information of IO tables for many economies, especially in a regional context, makes this "dynamic" approach often unfeasible, since intermediate observations are not available for all the

[14] Note that in the hypothetical case when with an infinite number of intermediate points, the interaction terms would vanish and there would be a unique decomposition form.

factors. In previous works (Fernández, 2004; Fernandez *et al.*, 2005), the use of additional information of some of the factors has been studied. This papers suggest a different perspective, considering the possibility of including intermediate observations not of the factors, but concerning the dependent or endogenous variable z.

To illustrate this idea in a simple way, let us continue with the simplest case with two factors x and y. Suppose that for an intermediate period t' we have collected the value $z(t')$, although $x(t')$ and $y(t')$ are both unknown. Obviously, this situation disables a two-stages decomposition like the represented in Figure 3, but the additional information $z(t')$ can be used somehow to obtain a decomposition different from the average of equations (7) and (8), which would be the most appropriate solution if no additional information is available. Applying the PB approach to this context, we will have the following temporal paths for factors x and y:

$$x(t) = x^0 + \Delta x t^{\theta_x};\ \forall\, \theta_x > 0 \tag{32}$$
$$y(t) = y^0 + \Delta y t^{\theta_y};\ \forall\, \theta_y > 0 \tag{33}$$

And, consequently:

$$z(t) = x(t)y(t) = x^0 y^0 + x^0 \Delta y t^{\theta_y} + \Delta x y^0 t^{\theta_x} + \Delta x \Delta y t^{(\theta_x+\theta_y)};\ \forall\, \theta_x, \theta_y > 0 \tag{34}$$

If we include a stochastic component ε_t that allows z to diverge from the deterministic path (34), we obtain[15]

$$z(t) = x^0 y^0 + x^0 \Delta y t^{\theta_y} + \Delta x y^0 t^{\theta_x} + \Delta x \Delta y t^{(\theta_x+\theta_y)} + \varepsilon_t;\ \forall\, \theta_x, \theta_y > 0 \tag{35}$$

Or, equivalently

$$\Delta z(t) = z(t) - z^0 = x^0 \Delta y t^{\theta_y} + \Delta x y^0 t^{\theta_x} + \Delta x \Delta y t^{(\theta_x+\theta_y)} + \varepsilon_t;\ \forall\, \theta_x, \theta_y > 0 \tag{36}$$

Equation (36) is a non-linear model with two parameters to be estimated. Defining $\theta_x = \mathbf{b}'\mathbf{p}_x = \sum_{m=1}^{M} b_m p_{xm}$ and $\theta_y = \mathbf{b}'\mathbf{p}_y = \sum_{m=1}^{M} b_m p_{ym}$ where $\mathbf{p_x}$ and $\mathbf{p_y}$ are unknown probability distributions, equation (36) can be written as[16]

$$\Delta z(t) = x^0 \Delta y t^{\left(\sum_{m=1}^{M} b_m p_{ym}\right)} + \Delta x y^0 t^{\left(\sum_{m=1}^{M} b_m p_{xm}\right)} + \Delta x \Delta y t^{\left(\sum_{m=1}^{M} b_m p_{xm} + \sum_{m=1}^{M} b_m p_{ym}\right)} + \sum_{j=1}^{J} v_j w_{tj} \tag{37}$$

Hence, it is possible to apply the Maximum Entropy estimation technique for non-linear relationships analyzed in the previous section. Upon having estimated both parameters, it is

---

[15] We assume that $\varepsilon_t = 0$ in the final period. This ensures that $z(t)$ has value z^1 in this period.

[16] Note that in (41) the stochastic term ε_t is also defined in the same way as equation (29).

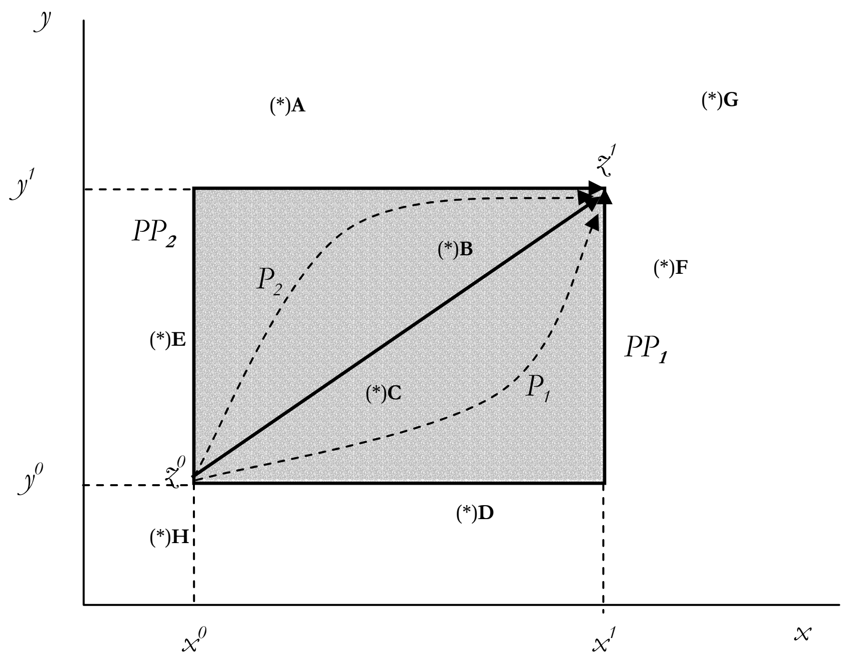
immediate to obtain the estimated respective contributions of changes in the determinants by substituting their values in the following equations

$$\Delta x\ Effect = \Delta x y^0 + \frac{\theta_x}{\theta_x + \theta_y} \Delta x \Delta y \tag{39a}$$

$$\Delta y\ Effect = x^0 \Delta y + \frac{\theta_y}{\theta_x + \theta_y} \Delta x \Delta y \tag{39b}$$

which are the reduced versions for the two-factors case of equation (18). The use of the framework outlined above can cause nontrivial problems if observations for intermediate periods of $z(t)$ are rather unlikely to be generated by a time path belonging to the class of paths defined by equation (38). We deal with such observations by fitting the most appropriate monotonic paths. Figure 4 depicts the possible situations when intermediate observations of z are available.

Figure 4. Estimated temporal paths with intermediate observations of z



For each intermediate period *t'* observations for z (marked with asterisks in Figure 4) can be categorized as points from A to H, depending on whether they are above or below the linear path and inside or outside the rectangle. If the observations are like B or C for every intermediate period no problems are encountered. However, the observations that depict situations that can not fit the monotonic paths of equations (32) and (33) are more problematic. If we observe points like A (conversely, E), this is given by a situation where $y(t')>y^1$ ($x(t')<x^0$) and the closest monotonic path is PP_2, which corresponds to a situation where $\frac{\theta_x}{\theta_y} \to \infty$. Points like D (F) are

caused by scenarios where $y(t')<y^0$ ($x(t')>x^1$), being PP_1 the most appropriate path and $\frac{\theta_y}{\theta_x} \to \infty$.

The most problematic observations of $z(t')$ are given by points like G or H: in fact, because these values are respectively greater than z^1 and smaller than z^0, these observations are not useful at all for the estimation procedure. Although we could approximate these points by respectively z^1 and z^0, there would not be evidences about the most appropriate monotonic path because all of them would be equally feasible (note that from z^0 depart an infinite number of monotonic paths and, equally, an infinite number of monotonic paths reach z^1).

6. Illustration: Decomposition of Sectoral Workforce Requirements in the Spanish Economy

We apply the technique developed in the previous sections to study the contributions of three determinants to changes in real sectoral labor requirements in Spain, over the period 1986-1994[17]. The required data were taken from 21-sector input-output tables for these years, expressed in constant prices of 1986. The intermediate blocks of the tables contain domestic deliveries only. Appendix A contains detailed information about how we treated the basic data to arrive at the data used in the analysis outlined below. The starting point is an input-output model that expresses the vector ($k \times 1$) of sectoral labor requirements **z** as the product of three factors, i.e. labor requirements per unit of gross output **u** (included as a diagonal $k \times k$ matrix), the Leontief inverse matrix **L** and the vector of final demands **f**.

$$\mathbf{z} = \hat{\mathbf{u}}\mathbf{L}\mathbf{f} \tag{40}$$

and the objective is to decompose the total change $\Delta\mathbf{z}$ into the following three components:

$$\mathbf{z}^{94} - \mathbf{z}^{86} = \Delta\mathbf{z} = \Delta\mathbf{u} \text{ effect} + \Delta\mathbf{L} \text{ effect} + \Delta\mathbf{f} \text{ effect} \tag{41}$$

We assume the following temporal paths for the elements of the factors:

$$u_k(t) = u_k^{86} + \Delta u_k t^{\theta_{u_k}} \text{ ; } k=1, ..., 21 \tag{42a}$$

$$l_{kj}(t) = l_{kj}^{86} + \Delta l_{kj} t^{\theta_{l_{kj}}} \text{ ; } k=1, ..., 21; j=1, ..., 21 \tag{42b}$$

$$f_k(t) = f_k^{86} + \Delta f_k t^{\theta_{f_k}} \text{ ; } k=1, ..., 21 \tag{42c}$$

According to equation (21), the contributions of changes in elements of **u**, **L** and **f** to the changes in **c** can be written as[18]

[17] It should be emphasized that the aim of this section is not so much to provide a "deep" analysis of the dynamics of Spanish labor requirements, but rather to provide an illustration of the methods proposed in this paper.

[18] The symbol $\circ$ indicates element-by-element (Hadamard) multiplication.

$$\Delta\mathbf{u}\,\mathit{effect} = \Delta\hat{\mathbf{u}}\mathbf{L}^{86}\mathbf{f}^{86} + \left[\boldsymbol{\Theta}^{\mathbf{u}}_{\mathbf{u+L}} \circ \Delta\hat{\mathbf{u}}\Delta\mathbf{L}\right]\mathbf{f}^{86} + \left[\boldsymbol{\Theta}^{\mathbf{u}}_{\mathbf{u+f}} \circ \Delta\hat{\mathbf{u}}\mathbf{L}^{86}\right]\Delta\mathbf{f} + \left[\boldsymbol{\Theta}^{\mathbf{u}}_{\mathbf{u+L+f}} \circ \Delta\hat{\mathbf{u}}\Delta\mathbf{L}\right]\Delta\mathbf{f} \tag{43a}$$

$$\Delta\mathbf{L}\,\mathit{effect} = \hat{\mathbf{u}}^{86}\Delta\mathbf{L}\mathbf{f}^{86} + \left[\boldsymbol{\Theta}^{\mathbf{L}}_{\mathbf{u+L}} \circ \Delta\hat{\mathbf{u}}\Delta\mathbf{L}\right]\mathbf{f}^{86} + \left[\boldsymbol{\Theta}^{\mathbf{L}}_{\mathbf{L+f}} \circ \hat{\mathbf{u}}^{86}\Delta\mathbf{L}\right]\Delta\mathbf{f} + \left[\boldsymbol{\Theta}^{\mathbf{L}}_{\mathbf{u+L+f}} \circ \Delta\hat{\mathbf{u}}\Delta\mathbf{L}\right]\Delta\mathbf{f} \tag{43b}$$

$$\Delta\mathbf{f}\,\mathit{effect} = \hat{\mathbf{u}}^{86}\mathbf{L}^{86}\Delta\mathbf{f} + \left[\boldsymbol{\Theta}^{\mathbf{f}}_{\mathbf{u+f}} \circ \Delta\hat{\mathbf{u}}\mathbf{L}^{86}\right]\Delta\mathbf{f} + \left[\boldsymbol{\Theta}^{\mathbf{f}}_{\mathbf{L+f}} \circ \hat{\mathbf{u}}^{86}\Delta\mathbf{L}\right]\Delta\mathbf{f} + \left[\boldsymbol{\Theta}^{\mathbf{f}}_{\mathbf{u+L+f}} \circ \Delta\hat{\mathbf{u}}\Delta\mathbf{L}\right]\Delta\mathbf{f} \tag{43c}$$

with the matrices $\boldsymbol{\Theta}$ defined as

$$\boldsymbol{\Theta}^{u}_{u+L} \equiv \begin{bmatrix} \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{l_{11}}} & \cdots & \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{l_{1K}}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{l_{K1}}} & \cdots & \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{l_{KK}}} \end{bmatrix} \quad \boldsymbol{\Theta}^{u}_{u+f} \equiv \begin{bmatrix} \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{f_1}} & \cdots & \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{f_1}} & \cdots & \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{f_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{u}_{u+L+f} \equiv \begin{bmatrix} \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{l_{11}}+\theta_{f_1}} & \cdots & \frac{\theta_{u_1}}{\theta_{u_1}+\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{u_K}}{\theta_{u_K}+\theta_{l_{KK}}+\theta_{f_K}} \end{bmatrix}$$

$$\boldsymbol{\Theta}^{L}_{u+L} \equiv \begin{bmatrix} \frac{\theta_{l_{11}}}{\theta_{u_1}+\theta_{l_{11}}} & \cdots & \frac{\theta_{l_{1K}}}{\theta_{u_1}+\theta_{l_{1K}}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{l_{K1}}}{\theta_{u_1}+\theta_{l_{K1}}} & \cdots & \frac{\theta_{l_{KK}}}{\theta_{u_K}+\theta_{l_{KK}}} \end{bmatrix} \quad \boldsymbol{\Theta}^{L}_{L+f} \equiv \begin{bmatrix} \frac{\theta_{l_{11}}}{\theta_{l_{11}}+\theta_{f_1}} & \cdots & \frac{\theta_{l_{1K}}}{\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{l_{K1}}}{\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{l_{KK}}}{\theta_{l_{KK}}+\theta_{f_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{L}_{u+L+f} \equiv \begin{bmatrix} \frac{\theta_{l_{11}}}{\theta_{u_1}+\theta_{l_{11}}+\theta_{f_1}} & \cdots & \frac{\theta_{l_{1K}}}{\theta_{u_1}+\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{l_{K1}}}{\theta_{u_K}+\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{l_{KK}}}{\theta_{u_K}+\theta_{l_{KK}}+\theta_{f_K}} \end{bmatrix}$$

$$\boldsymbol{\Theta}^{f}_{u+f} \equiv \begin{bmatrix} \frac{\theta_{f_1}}{\theta_{u_1}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{u_1}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{f_1}}{\theta_{u_K}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{u_K}+\theta_{f_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{f}_{L+f} \equiv \begin{bmatrix} \frac{\theta_{f_1}}{\theta_{l_{11}}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{f_1}}{\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{l_{KK}}+\theta_{f_K}} \end{bmatrix} \quad \boldsymbol{\Theta}^{f}_{u+L+f} \equiv \begin{bmatrix} \frac{\theta_{f_1}}{\theta_{u_1}+\theta_{l_{11}}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{u_1}+\theta_{l_{1K}}+\theta_{f_K}} \\ \vdots & \ddots & \vdots \\ \frac{\theta_{f_1}}{\theta_{u_K}+\theta_{l_{K1}}+\theta_{f_1}} & \cdots & \frac{\theta_{f_K}}{\theta_{u_K}+\theta_{l_{KK}}+\theta_{f_K}} \end{bmatrix}$$

As argued before, assuming that parameters θ are the same for all the factors (i.e., $\theta_{u_k} = \theta_{l_{kj}} = \theta_{f_k}$) for all j and k would yield Sun's (1998) solution. This would be a natural thing to do if no information would be available for the years in-between 1986 and 1994. To illustrate the techniques outlined in the previous sections, we will estimate some of the θ parameters by employing additional information of the sectoral labor requirements **z** in two intermediate years: 1989 and 1992. First, we had to decide on the values to be assigned to the a priori distributions contained in the support vector **b** and the possible realizations for the random term in vectors **v**. The following vectors were used for all parameters throughout the empirical analyses below:[19]

b=[-5.0, -3.0, -1.0, 1.0, 3.0, 5.0, 7.0]’ and **v**=[-100, -50, 0, 50, 100]’

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19 In the empirical application we have been forced to make a couple of exceptions: due to the extremely great labor requirements observed for sectors 14 and 21 (in relative terms to the rest of the sectors), it was necessary to consider wider bound for the vector **v** in order to get feasible estimates. In these two cases, the bounds were enlarged and go from –150 to 150. Golan *et al.* (1996, p. 138) asserted that the estimation results are generally not very sensitive to the choice of a particular set.

Note that to the central value of **b**, namely b^*, is assigned a value 1 since the solution of the constrained maximization problem (36) without additional information yields estimates $\hat{\theta}$ equal to this value b^*. This means that, with lack of information for a temporal path, there are not reasons to assume a convex (i. e., $\hat{\theta} < 1$) or concave (i. e., $\hat{\theta} > 1$) path either. Let us define $\theta_{uk} = \mathbf{b}'\mathbf{p}_{uk} = \sum_{m=1}^{M} b_m p_{ukm}$, $\theta_{lkj} = \mathbf{b}'\mathbf{p}_{lkj} = \sum_{m=1}^{M} b_m p_{lkj_m}$ and $\theta_{fk} = \mathbf{b}'\mathbf{p}_{fk} = \sum_{m=1}^{M} b_m p_{fk_m}$ where $\mathbf{p}_{uk}$, $\mathbf{p}_{lkj}$ and $\mathbf{p}_{fj}$ are unknown probability distributions. In an equivalent way as that depicted previously for the simplest two-factors case, the sectoral labor use in a sector i will be:

$$z_i(t) = u_i^{86}\sum_{k=1}^{21} l_{ik}^{86} f_k^{86} + u_i^{86}\sum_{k=1}^{21}\Delta l_{ik} f_k^{86} t^{\theta_{lkj}} + \Delta u_i \sum_{k=1}^{21} l_{ik}^{86} f_k^{86} t^{\theta_k} + u_i^{86}\sum_{j=1}^{21} l_{ik}^{86}\Delta f_k t^{\theta_{fk}} +$$
$$\Delta u_i \sum_{k=1}^{21}\Delta l_{ik} f_j^{86} t^{(\theta_{ui}+\theta_{lik})} + \Delta u_i \sum_{k=1}^{21} l_{ik}^{86}\Delta f_k t^{(\theta_{ui}+\theta_{fk})} + u_i^{86}\sum_{k=1}^{21}\Delta l_{ik}\Delta f_k t^{(\theta_{lik}+\theta_{fk})} + \tag{44}$$
$$\Delta u_i \sum_{k=1}^{21}\Delta l_{ik}\Delta f_k t^{(\theta_{ui}+\theta_{lik}+\theta_{fk})} + \sum_{h=1}^{H} v_h w_{th}$$

In (44) has already been included an stochastic element (last term) as we commented before. In an equivalent way as that depicted in program (38) for the simplest two-factors case, the following GME program has to be solved, in which this expression (44) ends up as a constraint

$$\underset{\mathbf{p}_u,\mathbf{p}_L,\mathbf{p}_f,\mathbf{w}}{Max}\; H(\mathbf{p}_u,\mathbf{p}_L,\mathbf{p}_f,\mathbf{w}) = -\sum_{k=1}^{21}\sum_{j=1}^{21}\sum_{m=1}^{M}\left[p_{ukm}\, ln(p_{ukm}) + p_{lkj_m}\, ln(p_{lkj_m}) + p_{fk_m}\, ln(p_{fk_m})\right] - \sum_{t=1}^{T}\sum_{h=1}^{H} w_{th}\, ln(w_{th}) \tag{45}$$

subject to:

$$\sum_{m=1}^{M} p_{ukm} = \sum_{m=1}^{M} p_{lkj_m} = \sum_{m=1}^{M} p_{fk_m} = 1,\ \forall k, j = 1,\dots,21$$
$$\sum_{h=1}^{H} w_{th} = 1,\ \forall t = 1,\dots,T$$
$$z_i(t) = u_i^{86}\sum_{k=1}^{21} l_{ik}^{86} f_k^{86} + u_i^{86}\sum_{k=1}^{21}\Delta l_{ik} f_k^{86} t^{\theta_{lkj}} + \Delta u_i \sum_{k=1}^{21} l_{ik}^{86} f_k^{86} t^{\theta_k} + u_i^{86}\sum_{j=1}^{21} l_{ik}^{86}\Delta f_k t^{\theta_{fk}} +$$
$$\Delta u_i \sum_{k=1}^{21}\Delta l_{ik} f_j^{86} t^{(\theta_{ui}+\theta_{lik})} + \Delta u_i \sum_{k=1}^{21} l_{ik}^{86}\Delta f_k t^{(\theta_{ui}+\theta_{fk})} + u_i^{86}\sum_{k=1}^{21}\Delta l_{ik}\Delta f_k t^{(\theta_{lik}+\theta_{fk})} +$$
$$\Delta u_i \sum_{k=1}^{21}\Delta l_{ik}\Delta f_k t^{(\theta_{ui}+\theta_{lik}+\theta_{fk})} + \sum_{h=1}^{H} v_h w_{th},\ \forall i = 1,\dots,21;\ \forall t = 1,\dots,T$$

The estimates obtained here will be used to obtain the respective contributions for the effect of changes in the three factors considered.

6.1 Contributions of changes in the determinants

Table 1 reports the values of the variation in the sectoral workforce use between 1986 and 992 and the intermediate observations of this variable in 1989 and 1992; being these data obtained form the Spanish National Accounts (INE, 1990 and 1993). These observations lead to the estimates of the parameters that characterize the temporal paths (42a-42c) and they are shown in Appendix B. Table 1 also reports the results of the decomposition for changes in the three determinants, as well as ratios that compare the results with those obtained by average decompositions:

Table 1: Variation in z between 1986 and 1994, values of z(t) in 1989 and 1992 and decomposition results [a]

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Sector	***Δz***[b]	***z(1989)***	***z(1992)***	***Δu Eff***	***ΔL Eff***	***Δf Eff***	ρ_u^F	ρ_L^F	ρ_f^F	ρ_u^P	ρ_L^P	ρ_f^P
s1	-543	1488	1212	-652.31	-255.23	364.54	107.21	105.58	118.67	107.75	103.00	117.51
s2	-34	150	133	-42.81	-23.04	31.85	102.78	105.12	107.71	103.42	102.70	106.77
s3	-18	84	85	-16.79	-45.06	43.85	99.74	104.71	104.74	103.31	101.96	103.31
s4	2	183	188	-36.27	-26.50	64.77	108.33	112.26	109.62	109.05	110.22	109.22
s5	-1	144	139	-54.22	-17.68	70.90	101.13	106.17	102.36	101.86	101.48	101.79
s6	25	308	296	-47.56	-19.59	92.16	105.58	107.91	104.48	106.00	105.83	104.27
s7	18	115	120	0.67	-1.65	18.99	101.16	102.40	100.16	101.19	102.43	100.17
s8	5	207	207	-80.57	-11.26	96.84	103.45	109.50	103.94	103.82	103.97	103.63
s9	-20	269	265	-173.21	-13.07	166.28	103.88	109.86	104.82	104.20	101.14	104.48
s10	22	425	425	-39.06	-13.60	74.67	103.67	105.52	102.89	103.77	104.93	102.84
s11	-80	432	393	-274.73	10.89	183.85	101.79	136.69	101.05	101.78	135.82	101.06
s12	25	167	183	-9.76	-5.54	40.30	105.41	106.97	102.19	105.52	106.58	102.16
s13	23	387	390	43.88	-62.81	41.93	104.39	106.35	104.76	105.19	107.52	105.61
s14	238	1139	1206	-5.55	-14.83	258.38	112.72	107.01	100.62	112.77	106.98	100.62
s15	336	1998	2119	-133.53	61.93	407.60	106.12	102.97	101.47	105.96	102.30	101.52
s16	188	816	831	58.82	4.78	124.40	107.43	102.05	96.76	107.39	102.82	96.75
s17	10	565	574	-175.81	25.02	160.79	100.83	103.11	100.43	100.58	99.62	100.69
s18	31	135	151	-40.52	28.59	42.94	104.21	106.83	99.56	103.22	103.93	100.43
s19	23	303	321	-33.95	-55.73	112.69	102.21	102.30	101.80	103.43	100.85	101.44
s20	146	299	353	54.78	30.96	60.27	107.49	105.35	91.79	106.88	107.46	91.39
s21	860	3014	3411	-108.75	34.43	934.32	110.14	99.52	101.10	110.06	99.10	101.11
Total	**1256**	**12628**	**13002**	**-1767.28**	**-369.02**	**3392.30**	**104.69**	**106.48**	**103.09**	**104.99**	**103.63**	**102.94**

[a] Values of columns (1-6) measured in thousand of workers.

[b] Columns (4-6) do not always add up to the numbers in column (1) due to rounding.

Column (1) reports the actual change in sectoral labor requirements in the 21 Spanish sectors between 1986 and 1994. Labor requirements increased in all but six sectors, “Agriculture” (1), “Energy” (2), “Mining products” (3), “Chemical products” (5), “Transport equipment” (9) and “Textiles” (11). Columns (2)-(4) present the results of the decomposition analysis applying the PB approach with the additional information considered. The values for the Spanish economy as a whole in the bottom row are obtained by simply adding the sectoral results. Clearly, declining labor requirements per unit of gross output would have led to lower workforce use in most sectors if nothing else would have changed (the $\Delta\mathbf{u}$ effect is generally negative). The generally negative results for the $\Delta\mathbf{L}$ effect suggest that the domestic input coefficients have changed in such a way that labor requirements would have decreased, in the absence of changes in the level and composition of final demand. This result can be given by changes in technology (technological progress or substitution of inputs) or by variations in the trade pattern of Spain. Finally, the contributions of the $\Delta\mathbf{f}$ effect are positive for all sectors. This is not a very surprising result, since in the demand-driven input-output model, positive final demand growth (which has actually happened in the Spanish economy) will always yield increases in labor requirements, unless labor requirements per unit of output are reduced considerably and/or input coefficients cause substitution of inputs towards inputs with lower labor cost coefficients.

Besides the results of the decomposition, the figures of columns (7)-(12) are of most interest. Columns (7-9) present ratios ρ^F that compare the effect obtained by PB approach with those obtained by taking average over the 24 traditional decomposition formulae. A value of the ratio equal to 100 implies that the same result would be obtained by both approaches, in other words, the information included does not lead to results remarkably different from a "non-informative situation" where an average solution is the most appropriate solution. Columns (10-12) present an equivalent ratio ρ^P for a comparison of the PB technique with the mean of polar decompositions, since this approach is often used because it yields very similar results to the full mean over all decomposition forms with fewer computations being required.

As we can see from these columns, the deviations are sometimes considerable. Although in general terms the results obtained by PB approach are quite close to both average decompositions (see the values of the for the whole of sectors), for some specific sectors the contributions of the effects vary remarkably. The results for changes in the final demand "Agriculture" (1) are a good example, because if we apply the PB approach to this sector we will obtain that the effect of variations in its final demand is around 18% greater than the result obtained by an average decomposition. "Building materials and construction" (14) or "Textiles" (11) are other examples of great variations for the effects of respectively, changes in labor use per unit of output and Leontief matrix.

Roughly speaking, the most substantial deviations are found for the sectors for which the estimated parameters deviate strongly from one, the value implicitly chosen when taking averages (see Appendix C). However, some exceptions to this generic rule can be found, due to the matricial nature of the decomposition at hand. Consequently, a estimated time path for one of the factors in a sector i (let's say the final demand for commodities produced by this sector)that differs greatly from one, can well affect the contribution of changes in other factor for other sector j (for example, the labor requirements per unit of output in sector j) although its corresponding estimated path does not differ remarkably form a linear one.

6.2. Comparison to an annual decomposition

The above empirical application illustrates how the PB approach can use limited information of the dependent variable to obtain unique decompositions. Outcomes of Table 1 showed that the results were remarkably different from averages decompositions for some specific cases. This subsection is devoted to test if the outcomes of PB decomposition are somehow more accurate that the alternative of using those pragmatic average solutions..

The additional information considered in the empirical application were data of sectoral labor use in 1989 and 1992, but this was made just to exemplify how the proposed PB technique worked. In fact, the Spanish Statistical Institute provides annual information of the three factors considered from 1986 to 1994, which gives the possibility of making an annual decomposition. This dynamic approach reduces the gravity of the non-uniqueness problem (see Figure 3) although needs of more data than the commented PB approach. The following table takes as reference the contributions of the determinants obtained by an annual average decomposition and compares with them the outcomes obtained by the full mean of decompositions, the mean of polar decompositions (being both computed only with the initial and final data) and the PB approach with the additional information considered.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Sector	***Δu Eff***	***ΔL Eff***	***Δf Eff***	e_u^F	e_L^F	e_f^F	e_u^P	e_L^P	e_f^P	e_u^{PB}	e_L^{PB}	e_f^{PB}
s1	-697.08	-277.84	431.92	7856.63	1303.30	15559.79	8402.16	903.00	14814.12	2004.65	511.23	4540.56
s2	-48.68	-23.94	38.63	49.46	4.09	81.98	53.17	2.26	77.37	34.53	0.81	45.91
s3	-16.01	-45.92	43.94	0.67	8.34	4.29	0.06	2.98	2.22	0.60	0.74	0.01
s4	-39.46	-27.61	69.06	35.76	15.99	99.56	38.42	12.68	95.25	10.17	1.22	18.44
s5	-52.65	-19.86	71.51	0.92	10.26	5.03	0.33	5.92	3.46	2.45	4.74	0.37
s6	-50.52	-20.41	95.93	29.91	5.08	59.63	31.89	3.60	56.91	8.73	0.67	14.23
s7	4.90	-1.04	14.14	18.00	0.33	23.21	18.00	0.33	23.21	17.94	0.38	23.51
s8	-82.86	-12.38	100.24	24.76	4.39	50.00	27.56	2.39	46.20	5.23	1.25	11.59
s9	-177.18	-12.40	169.59	109.15	0.26	120.04	120.12	0.27	109.08	15.80	0.44	10.96
s10	-36.76	-16.10	74.87	0.83	10.29	5.27	0.77	9.84	5.10	5.27	6.23	0.04
s11	-273.57	5.97	187.59	13.41	3.96	31.95	13.23	4.17	32.24	1.36	24.13	14.02
s12	-9.08	-8.01	42.09	0.03	8.03	7.05	0.03	7.92	7.00	0.46	6.11	3.22
s13	50.97	-62.47	34.49	79.99	11.62	30.62	85.83	16.41	27.18	50.38	0.12	55.35
s14	-5.89	-16.92	260.82	0.94	9.41	16.28	0.94	9.38	16.27	0.12	4.39	5.94
s15	-79.13	57.66	357.47	2180.73	6.21	1954.13	2199.14	8.33	1936.78	2959.89	18.30	2512.75
s16	118.02	4.12	65.86	4002.75	0.31	3932.28	4000.54	0.28	3934.47	3504.73	0.43	3427.55
s17	-170.23	22.78	157.45	17.18	2.22	7.05	20.89	5.47	4.98	31.18	5.04	11.15
s18	-29.68	27.11	33.57	84.81	0.12	91.37	91.83	0.16	84.36	117.68	2.19	87.78
s19	-17.69	-73.54	114.23	241.21	363.22	12.44	229.20	333.97	9.83	264.59	317.01	2.37
s20	74.75	30.67	40.59	565.71	1.64	628.29	552.03	3.46	642.88	398.75	0.08	387.25
s21	92.76	42.96	724.28	36673.40	69.94	39946.39	36701.85	67.48	39916.72	40608.02	72.73	44117.96
Total	**-1445.07**	**-427.19**	**3128.26**	**228.00**	**42.88**	**250.33**	**229.32**	**37.42**	**248.69**	**223.70**	**31.28**	**235.14**

Columns (1)-(3) present the results of the average annual decomposition analysis; again the last row presents the results for the whole of the Spanish economy. The following columns report square differences for each effect of the full mean of decompositions (columns 4–6), the mean of polar decompositions (columns7-9) and the PB approach (columns 10-12), respectively marked with superscripts *F*, *P* and *PB*. Note that, the results obtained by the PB technique are the closest to the annual decomposition for each of the three effects. The conclusion would be that, using only a limited amount of additional information (data of sectoral labour use in 1989 and 1992), it is possible to obtain more similar results to an annual SDA than traditional average solutions

7. Conclusions

A well known problem of SDA is that the results often depend strongly on the specific decomposition formula chosen, whereas numerous formulae are equivalent from a theoretical point of view. This non-uniqueness problem is often solved rather pragmatically, by reporting an average of (a subset of) all possible formulae. This paper applies a decomposition methodology using Generalized Maximum Entropy (GME) econometrics to select the decomposition formula that provides an optimal 'fit' to additional empirical information. The point of departure is the "path based" (PB) method proposed in previous work, showing that one specific solution of this technique is equivalent to taking the average over all traditional decomposition formulae. Since the solutions of this method depend on unknown parameters that characterize the paths of the determinants, these parameters can be estimated even if the available data is very limited. If information about the values of the dependent variable is available for intermediate periods between the initial period and the final period of the analysis, a non-linear GME program can be solved to estimate them and obtain an unique decomposition.

We applied the methodology to quantify the contributions of three determinants of changes in sectoral labor requirements in Spain between 1986 and 1994, i.e. labor requirements per unit of gross output, input coefficients and final demand levels. As additional information, data of sectoral workforce use in 1989 and 1992 were included. The results indicate that, firstly, the use of additional information in the PB approach can well yield results that differ substantially from the mean over all traditional decomposition formulae. Secondly, an annual average decomposition was also obtained to take it as a reference for a comparison, showing that the PB approach yielded in this case closer results than more traditional average solutions. These result lead us to believe that the PB method provides an interesting alternative to computing averages over decomposition formulae.

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Appendix A: Construction of Data for Empirical Illustration

This appendix depicts the main sources of information consulted for the empirical study in Section 6, as well as the manipulations we had to carry out before we could apply the decomposition analysis to changes in sectoral labor requirements in Spain. The original tables for 1986 and 1994 had a sectoral classifications distinguishing 57 sectors. For the sake of simplicity in the paper we have preferred to work with a less detailed classification and we have aggregated them until considering only 21 sectors (see Table A.1). This same aggregation was made with the data of sectoral labor requirements for the initial and final periods, as well as the data used as additional information in 1989 and 1992:

Table A.1: Sector classification

Sector	**Name**	**Equivalence to R57**
s1	Agriculture	1
s2	Energy	2+3+4+5+6+7+8+9+10+11
s3	Minerals and mining products	12+13
s4	Non-metallic products	14+15+16+17
s5	Chemical products	18
s6	Metallic products excepting transport equipment	19
s7	Machinery for agriculture and industry	20
s8	Office equipment, measuring equipment and others	21+22
s9	Transport equipment	23+24
s10	Food, drinks and tobacco	25+26+27+28+29
s11	Textiles, leather and clothing	30+31
s12	Paper and derived products	33+34
s13	Industries not elsewhere classified	32+35+36
s14	Building materials and construction	37
s15	Commerce an repairing services	38+39
s16	Restaurants, hotels and cafes	40
s17	Transport services	41+42+43+44+45
s18	Communications	46
s19	Finance and insurance	47+48
s20	Real estate and services to companies	49+50
s21	Other services	51+52+53+54+55+56+57

Appendix B: Estimates of the parameters

The values that appear in the following tables have been obtained using GAMS and the CONOPT algorithm to non-linear optimisation problems.

Table B.1: estimates for θ_u and θ_f

Sector	$\hat{\theta}_{uk}$	$\hat{\theta}_{fk}$
s1	0.632	0.000
s2	0.970	0.970
s3	0.635	1.005
s4	1.244	0.802
s5	0.632	0.791
s6	1.385	0.434
s7	0.993	0.897
s8	1.213	0.710
s9	0.999	0.782
s10	0.865	0.459
s11	0.800	0.752
s12	1.252	0.404
s13	0.526	1.309
s14	1.525	0.000
s15	2.723	0.506
s16	1.005	0.000
s17	0.525	0.821
s18	0.814	1.059
s19	0.799	0.996
s20	0.700	0.000
s21	2.903	0.650

Table B.2: estimates for θ_1

$\hat{\theta}_{lkj}$	s1	s2	s3	s4	s5	s6	s7	s8	s9	s10	s11	s12	s13	s14	s15	s16	s17	s18	s19	s20	s21
s1	0.979	0.999	0.999	1.000	0.984	0.999	0.999	0.994	0.971	0.815	0.992	0.995	0.991	0.989	0.992	0.877	0.995	1.000	0.997	0.997	0.974
s2	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	0.997	0.999	0.998	0.999	1.000	1.000	0.999	1.000
s3	0.998	0.997	0.916	0.998	0.991	0.947	0.975	0.945	0.801	0.987	0.997	0.999	0.994	0.874	0.979	0.986	0.990	1.000	0.999	0.990	0.964
s4	1.000	1.001	0.998	0.988	1.010	1.000	1.000	1.002	1.011	1.005	1.002	1.000	1.001	1.100	1.007	1.011	1.002	1.000	1.002	1.010	1.006
s5	0.999	1.000	1.000	1.000	0.990	0.999	1.000	0.997	0.992	0.994	0.978	1.001	1.000	0.987	0.994	0.988	0.999	1.000	0.999	0.998	0.997
s6	0.991	1.002	0.992	0.998	1.003	1.005	1.004	0.999	1.080	0.991	0.973	0.999	1.012	1.044	1.018	1.025	1.010	1.000	1.003	1.008	1.057
s7	0.991	1.001	0.994	0.997	0.996	0.997	1.001	0.991	1.004	0.990	0.999	0.999	0.998	1.004	1.001	1.002	1.001	1.000	1.000	1.003	1.048
s8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.003	1.006	1.000	1.000	1.000	1.000	1.002	1.001	1.000	1.001	1.000	1.000	1.000	1.004
s9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000	0.999	0.995	0.999	0.995	1.000	1.000	0.999	0.989
s10	1.012	1.000	1.000	1.000	0.999	1.000	1.000	1.000	0.999	1.004	0.994	1.000	0.999	0.999	0.999	0.953	1.000	1.000	1.000	1.000	1.000
s11	1.000	1.000	1.000	1.000	1.001	1.000	1.000	1.000	1.000	1.002	0.990	1.000	1.001	1.009	1.001	1.000	1.000	1.000	1.000	1.000	1.001
s12	0.987	1.003	0.997	0.995	0.980	0.998	0.995	1.000	1.009	0.949	0.994	1.071	1.010	1.026	1.054	1.032	1.002	1.006	1.020	1.024	1.001
s13	0.996	1.005	1.000	1.002	1.017	1.007	0.998	1.008	1.094	1.019	1.041	1.001	1.085	1.164	1.035	1.028	1.002	1.001	1.008	1.015	1.041
s14	0.926	1.021	0.979	0.970	0.927	0.985	0.980	0.987	0.965	0.864	1.001	0.980	1.019	1.026	1.323	1.408	1.094	1.009	1.216	1.715	0.926
s15	0.941	0.998	0.911	0.982	0.967	0.971	0.982	0.971	0.993	0.818	0.967	0.982	1.001	1.010	0.997	1.022	0.957	1.002	1.009	1.005	0.989
s16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.001	0.999	1.001	1.001	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.002
s17	1.006	0.999	1.001	1.001	1.002	1.001	1.001	1.001	1.002	1.010	1.003	1.001	1.000	0.999	1.000	0.997	1.008	0.999	0.999	0.999	1.006
s18	1.003	1.001	1.001	1.002	1.006	1.002	1.002	1.003	1.007	1.009	1.006	1.001	1.002	1.010	1.010	1.013	1.006	1.001	0.997	1.008	1.027
s19	1.006	0.997	1.001	1.001	1.004	0.999	0.999	0.998	1.000	1.004	0.994	1.001	1.000	0.935	0.981	0.968	0.998	1.000	0.844	1.008	1.006
s20	0.990	0.998	0.996	0.997	0.986	0.995	0.994	0.990	0.966	0.972	0.989	0.996	0.996	0.970	0.997	0.992	0.995	1.001	1.010	0.991	0.981
s21	0.997	1.000	0.996	0.997	0.973	0.997	0.997	0.980	0.991	0.990	0.996	0.998	0.996	1.006	1.010	1.002	0.987	1.004	1.010	1.175	0.754