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Final demand impact on output: a Macro Multipliers approach

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Draft paper

Abstract

In this paper the attempt is made to provide a method of analysis that can give a further insight into the interactions between output by industries and final demand by institutional sectors. An application that relies on a regional data base, inspired by the Social Accounting Matrix, illustrates how macro-multipliers ruling the multi-sector multi-industry interactions can be defined and evaluated. This feature greatly helps in showing the impact of the structure of macroeconomic variables since all the possible behavior of the economy are determined by those multipliers: either those patterns that have emerged, because have been activated by the actual shock, and those that have kept latent. The identification of Macro Multipliers allows for the consistent definition of Social Accounting Matrix multipliers, a tool especially efficient in the study of propagation since it is not confined to predetermined structures of macroeconomic variables.

Keywords: Structural Change, Multipliers Analysis, Social Accounting Matrix.

JEL classification: C67, D31, D57, R15

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1 Introduction

From the Social Accounting Matrix (SAM) approach a model of circular income flow which is more articulated than the usual one emerges: each macroeconomic flow variable, conveniently disaggregated, generates a second flow variable through the use of a structural matrix and progressively so on until the loop is closed. Final demands determine total outputs and value added by industry; the latter generates domestic incomes by factors which compose disposable incomes by institutional sectors; these give rise to final demands closing the loop (Miyazawa, 1970).

To address this progress in the design of a data base which provides meaningful sectorization of the major macroeconomic variables, flexible tools of analysis are needed, to get a deeper insight into the propagation phenomena characterizing sectoral and industrial interactions. In these phenomena the scale, but more especially, the structure of macroeconomic variables play a major role. The traditional tools for studying propagation are those provided by impact multipliers. These tools, however, provide procedures that do not give a complete account of the effects of the changing structures of macro-variables. Multiplier analysis is based on a particular structure of exogenous variable, for example final demand, that using in Input-Output model without considering the link among exogenous and endogenous variables.

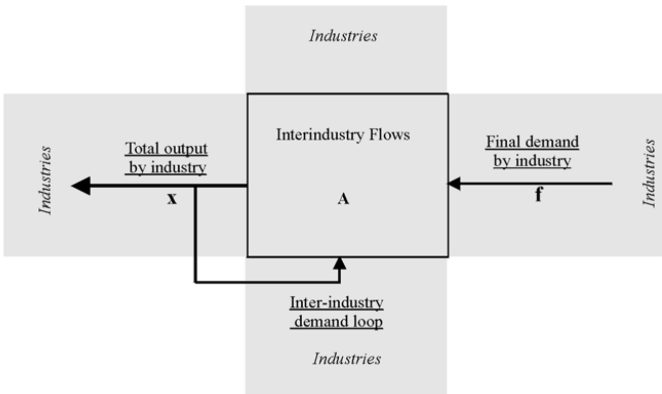
In our work we attempt to find which is the better composition of exogenous variable to obtain a particular effect on variable objective. The propagation analysis we propose is based on a decomposition that allows for the identification and quantitative determination of aggregated Macro Multipliers (MM), which lead the economic interactions, and the structures of macroeconomic variables, that either hide or activate these forces. The analysis will be applied to an extended final demand-output loop that can be quantitatively tested forwarding a shock on a given macro-variable and observing the effects on another macro-variable within the loop. It will identify the most efficient structure to the aim of the policy maker.

In section 2 the discussion on impact multipliers analysis is briefly referred. Section 3 shows the formalization of an extended circular flow loop on which the analysis will be performed. Section 4 shows the singular values decomposition related to eigenvalues decomposition and define MM approach. Section 5 the deterministic analysis of propagation is performed in order to identify and quantify all the MM that rule the economic interactions. Appendix A shows the Social Accounting Matrix used.

2 Multiplier analysis

The original Input-Output (I-O) problem is to search the output vector consistent with final demand vector for I-O sectors, given structural interrelation among industry sector. Since, in the following section, income will be disaggregated by institutional sectors, in order to avoid misinterpretation, we will use the term industries for producing sectors, and the sector for institutional sectors. Such a vector conveniently faces the predetermined final demand vector $\mathbf{f}$ by industries, and the induced industrial demand. The equilibrium output vector is

Figure 1: Inter-industry output flows



given by

$$\mathbf{x} = \mathbf{R} \cdot \mathbf{f} \tag{1}$$

where $\mathbf{R} = [\mathbf{I} - \mathbf{A}]^{-1}$ and $\mathbf{A}$ is the constant technical coefficients matrix, and generally exists, as in general the technology can be expected to be productive, i.e. the technology is such that a part of total output is still available for final uses, after the intermediate requirements have been satisfied. In this case, $\mathbf{A}$ satisfies the Hawkins-Simon conditions. The $\mathbf{R}$ matrix is usually referred to as the Leontief multipliers matrix (Leontief, 1965) and its elements, r_{ij}, show the direct and indirect requirements of industry output i per unit of final demand of product at industry j. Extensive use is made of matrix $\mathbf{R}$ within the traditional multipliers analysis is based on it. The $\mathbf{R}$ matrix provides, in fact, a set of disaggregated multipliers that are recognized to be the most precise and sensitive for studies of detailed economic impacts. These multipliers recognize the evidence that total impact on output will vary depending on which industries are affected by changes in final demand. The i^{th} total output multiplier measures the sum of direct and indirect input requirements needed to satisfy a unit final demand for goods produced by industry i. I-O multipliers can be derived from either an open I-O model and a partially closed I-O model. The first set includes type I and type II multipliers. For the determination of type I multiplier all components of final demand are treated exogenously. Type I multiplier will then represent the ratio of direct and indirect output changes to the initial direct change in final demand.

Multipliers can be however determined taking one or more components of final demand as endogenous. If the only final demand component to be treated endogenously is personal consumption expenditures, the multipliers are referred to as type II. In this case the model is said to be partially closed with respect to households. Each type II multiplier will then represent the ratio of the direct indirect and induced changes to the initial direct change. If another final demand component such as state and local government expenditure is also treated endogenously the multiplier is referred as type III (Lee, 1986).

When a final demand component is made endogenous the corresponding part of value added must also be treated endogenously; consequently, personal consumption expenditures have counterpart in value added referred to as wages and salaries; state and local government expenditures a counterpart referred to as taxes etc. The inverse coefficients of the augmented matrix reflect the induced effects of changed incomes on final outputs. Finally income and employment multipliers, type IV multipliers, can be obtained by pre-multiplying the matrix of output multipliers by a row vector of wage to output ratios in the case of income, and employment to output ratio in case of employment (Polenske and Jordan, 1988).

It has to be stressed, however, that all these measures, built starting from matrix $\mathbf{R}$, are not independent of structure of the either total output vector, neither which we observe the effects, nor of structure of final demand vector on which we impose the unit demand shock. The column sum of the $\mathbf{R}$ matrix in equation 1 implies the consideration of a set of final demand vectors of the type:

$$\mathbf{f}^1 = \begin{bmatrix} 1 \\ 0 \\ . \\ 0 \end{bmatrix}, \mathbf{f}^2 = \begin{bmatrix} 0 \\ 1 \\ . \\ 0 \end{bmatrix}, \cdots, \mathbf{f}^m = \begin{bmatrix} 0 \\ 0 \\ . \\ 1 \end{bmatrix} \qquad (2)$$

while the sum of row elements in equation 1 implies the consideration of a final demand structure of the type:

$$\mathbf{f} = \begin{bmatrix} 1 \\ 1 \\ . \\ 1 \end{bmatrix} \tag{3}$$

We can expect that these measures hold for demand vectors of varying scale but with the same structures of equations 2 or 3. However neither the demand vector nor its changes will ever assume a structure of this type. This is why some authors come to the drastic conclusion that "multipliers should be never used" (Skolka, 1986).

On the other hand it is a common opinion that the structure of final demand produces the most different effects on the level of total output (Ciaschini, 1989). Given a set of nonzero final demand vectors, whose elements sum up to a predetermined level, but with varying structures, we will have to expect that the corresponding level of total output will also vary considerably.

For these reasons we cannot confine our knowledge of the system to the picture emerging from measures which can only show what would happen if final demand assumed a predetermined and unlikely structure.

The similar approach can be used to performed multiplier analysis on extended I-O model which it is based on SAM framework. Also multiplier analysis on SAM model is subjected to the same criticism.

3 Extended Input-Output model

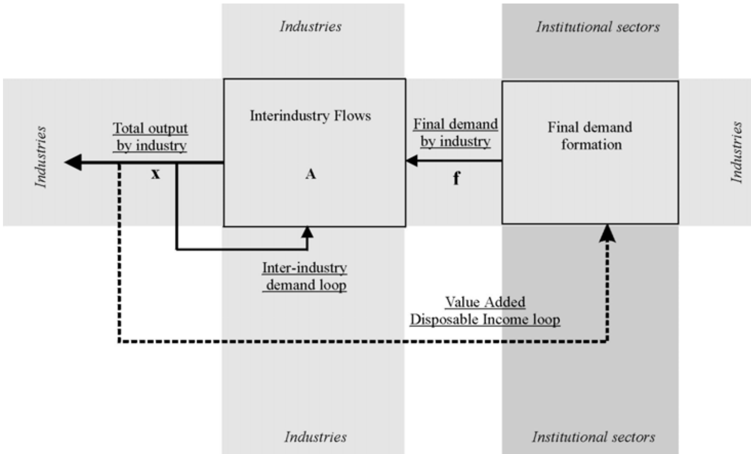
In our analysis we will use a multi-industry, multi-factor and multi-sector model following a Miyazawa approach (Miyazawa, 1970), figure 2, but in our case using extended income circular flow based on SAM scheme (Paytt, 2001). For the extended income-output model we adopt fixed prices and constant all coefficients and shares.

In fact, the results attained in social accounting encourage the attempt to build an extended version of the income circular flow where the interactions between industries and institutions could be specified and evaluated.

Figure 3 shows a diagram where the fundamental mechanism of production and distribution is shown in terms of interaction between industries, institutional sectors and primary factors (value added components).

In Figure 3 each arrow identifies an expenditure flow while each box a matrix transformation of a flow variable into another. In the upper part the inter industry demand loop in Figure 3 can be recognized.

Figure 2: Interindustry and intersectoral flows

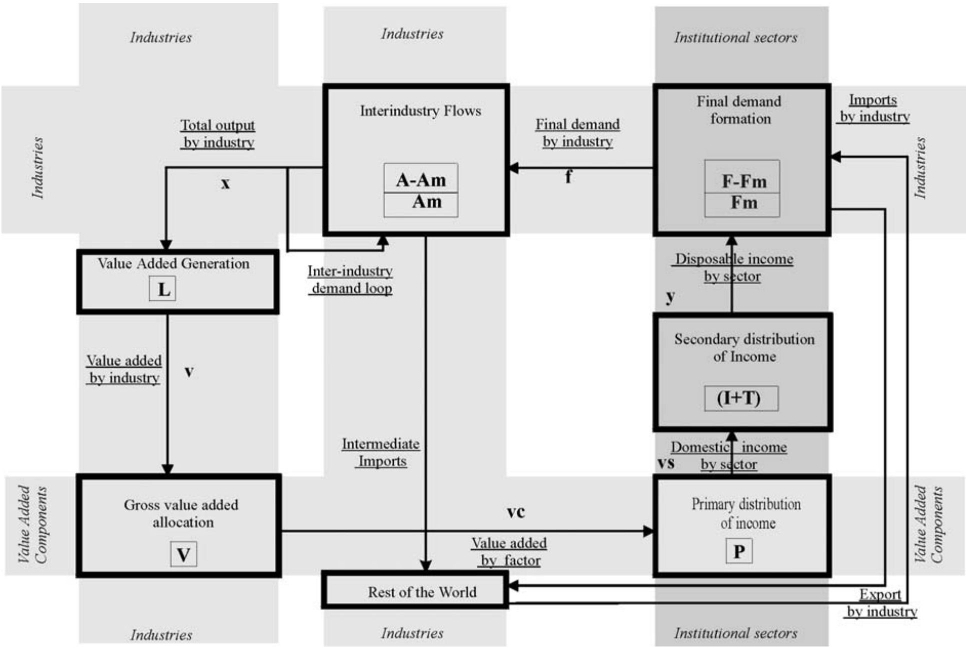


The extended output-income loop emerging in Figure 3 allows for an extension of the study of the propagation. We can choose, in fact, on which flow variable to act with a unit shock and on which variable to observe the effects. For each flow variable we need to specify an order of magnitude, such as the scale and a composition, or the structure. If we want to impose a unit shock on final demand and observe its propagation on domestic output we need to refer to equation 14, but other arrangements of structural matrices are easily found if we need to impose a shock on, say, income redistribution and observe it on value added by factor.

As shown in figure 3, with the dotted arrow, the income distribution process creates a feedback loop between industry output and final demand. This loop is built through various logical phases. The production process, that takes place at industry level, generates total output, $\mathbf{x}$, and gross value added by the m I-O industries, $\mathbf{v}(x)$, (Gross value added generation). Value added by I-O industry is then allocated to the c value added components (factors), $\mathbf{v}^c(x)$ (Gross value added allocation). Value added by components is then allocated to the s institutional sub-sectors, $\mathbf{v}^s(x)$ (Primary distribution of income). Value added by institutional sectors is then redistributed among them through taxation to generate disposable incomes by the s institutional sub-sectors, $\mathbf{y}(x)$ (Secondary distribution of income). Finally disposable income will generate final demand by institutional sub-sectors which will be transformed into final demand by I-O industries, $\mathbf{f}(x)$ (Final demand formation).

The extended I-O model starts from following fundamental equa-

Figure 3: extended output income circular flow



tion

$$\mathbf{x} + \mathbf{m} = \mathbf{B} \cdot \mathbf{i} + \mathbf{f} \tag{4}$$

where $\mathbf{x}$ is the output vector by industry, $\mathbf{m}$ is imports vector, matrix $\mathbf{B}$ is intermediates consumption and $\mathbf{f}$ finally is final demand vector. Our extended I-O model have a great part of final demand endogenous. For this raison we will determinate the our distributive structural matrices and this is topic for endogenous final demand analysis.

Gross value added generation(by industry)

$$\mathbf{v}(x) = \mathbf{L} \cdot \mathbf{x} \tag{5}$$

where $\mathbf{L}[m,m]$ gives the shares value added by industry starting from the output vector and technical coefficients matrix.

Gross value added allocation(by VA components)

$$\mathbf{v}^c(x) = \mathbf{V} \cdot \mathbf{v}(x) \tag{6}$$

where $\mathbf{V}[c,m]$ represents the distribution of value added to the factors (components).

Primary distribution of income(by Institutional sub-sectors)

$$\mathbf{v}^s(x) = \mathbf{P} \cdot \mathbf{v}^c(x) \tag{7}$$

where $\mathbf{P}[s,c]$ represents the distribution factors' value added income to the sectors.

Secondary distribution of income(by Institutional sub-sectors)

$$\mathbf{y}(x) = (\mathbf{I} + \mathbf{T}) \cdot \mathbf{v}^s(x) \quad (8)$$

where $\mathbf{T}[s,s]$ represents net income transfers among sub-sectors.

Final demand formation(by industry)

$$\mathbf{f}(x) = \mathbf{F}^0 \cdot \mathbf{y}(x) + \mathbf{K} \cdot \mathbf{y}(x) + \mathbf{f}^0 \quad (9)$$

where $\mathbf{F}^0$ provide the consumption demand structure by industry and is given by the product of two matrices, $\mathbf{F}^0 = \mathbf{F}^1 \cdot \mathbf{C}$, where $\mathbf{F}^1$ $[m,s]$ transforms the consumption expenditure by institutional sector into consumption by industry and $\mathbf{C}[s,s]$ represents the consumption propensities by institutional sector.

The matrix $\mathbf{K}$ represents the investment demand shares and is given by $\mathbf{K} = \mathbf{K1} \cdot s \cdot (\mathbf{I} - \mathbf{C})$ where $\mathbf{K1}[m,s]$ represents the investment demands to I-O industry and scalar s represents the share of private savings which is transformed into investment i.e. "active savings"; $\mathbf{f}^0$ is a vector of m elements which represents exogenous demand.

If we put $\mathbf{F} = [\mathbf{F}^0 + \mathbf{K}]$ equation 9 becomes

$$\mathbf{f}(x) = \mathbf{F} \cdot \mathbf{y}(x) + \mathbf{f}^0 \quad (10)$$

substituting through the equations 5-9 in 10 we get

$$\mathbf{f}(x) = \mathbf{F} \cdot [\mathbf{I} + \mathbf{T}] \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L} \cdot \mathbf{x} + \mathbf{f}^0 \quad (11)$$

We now turn to the output generation process shown in equation 4.

Output generation

$$\mathbf{x} + \mathbf{m} = \mathbf{A} \cdot \mathbf{x} + \mathbf{f}(x) \quad (12)$$

where $\mathbf{m}$ represents imports, $\mathbf{A}$ the technical coefficients matrix, $\mathbf{f}(x)$ represents the demand vector.

Imports can be modelled according its main components, intermediate consumptions, endogenous demand and exogenous demand:

Import

$$\mathbf{m} = \mathbf{A}^m \cdot \mathbf{x} + \mathbf{F}^m \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L} \cdot \mathbf{x} + \mathbf{f}^m \quad (13)$$

where $\mathbf{A}^m[m,m]$ represents the intermediate imports matrix, $\mathbf{F}^m[m,s]$ represents import shares of endogenous demands and $\mathbf{f}^m$ represents imports generated by an exogenous shock.

Substituting the equations 11 and 13 in 12 we finally get:

$$\mathbf{x} = [\mathbf{I} - (\mathbf{A} - \mathbf{A}^m) - (\mathbf{F} - \mathbf{F}^m) \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L}]^{-1} \cdot (\mathbf{f}^0 - \mathbf{f}^m) \quad (14)$$

4 Macro Multiplier approach: relationship between final demand and output

The direct and indirect effects of final demand on total output are then quantified in our model from structural matrix **R**.

$$\mathbf{R} = [\mathbf{I} - (\mathbf{A} - \mathbf{A}^m) - (\mathbf{F} - \mathbf{F}^m) \cdot (\mathbf{I} + \mathbf{T}) \cdot \mathbf{P} \cdot \mathbf{V} \cdot \mathbf{L}]^{-1} \tag{15}$$

The structural matrix[1] **R** of our model can be easily decomposed in a sum of m different tables through the singular value decomposition (Ciaschini, 1993).

The decomposition proposed can be applied both to square and to non-square matrices. Here the general case of square matrix **R** will be shown. The non-square matrix case is easily developed along the same lines.
To simplify we consider 2x2 model. Let us consider matrix **W** [2,2], for example, the square of matrix **R**:

$$\mathbf{W} = \mathbf{R}^T \cdot \mathbf{R}$$

Matrix **W** has a positive definite or semi definite square root. Given that $\mathbf{W} \geq 0$ by construction, its eigenvalues λ_i for $i = 1, 2$ shall be all real non negative (Lancaster and Tiesmenetsky, 1985).

The nonzero eigenvalues of matrices **W** and $\mathbf{W}^T$ coincide. The system of eigenvectors $[\mathbf{u}_i \; i = 1, 2]$ for **W** and $[\mathbf{v}_i \; i = 1, 2]$ for $\mathbf{W}^T$ are orthonormal basis.
We get then

$$\mathbf{R}^T \cdot \mathbf{u}_i = \sqrt{\lambda_i} \cdot \mathbf{v}_i \qquad i = 1, 2$$

We can construct the two matrices

$$\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2] \qquad \mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2]$$

As defined above, the eigenvalues of **W** coincide with singular values of **R** hence $s_i = \sqrt{\lambda_i}$ and we get

$$\mathbf{R}^T \cdot \mathbf{U} = [s_1 \cdot \mathbf{v}_1, s_2 \cdot \mathbf{v}_2] = \mathbf{V} \cdot \mathbf{S}$$

Structural matrix **R** in equation 15 can be then decomposed as

$$\mathbf{x} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^T \cdot \mathbf{f} \tag{16}$$

V is an [2,2] unitary matrix whose columns define the 2 reference structures for final demand:

$$\mathbf{v}_1 = \begin{bmatrix} v_{1,1} & v_{1,2} \end{bmatrix}$$

[1]Its numerical determination is shown in Table 1 section 5.

$$\mathbf{v}_2 = \left[\begin{array}{cc} v_{2,1} & v_{2,2} \end{array} \right]$$

$\mathbf{U}$ is an [2,2] unitary matrix whose columns define 2 reference structures for output:

$$\mathbf{u}_1 = \left[\begin{array}{c} u_{1,1} \\ u_{2,1} \end{array} \right], \mathbf{u}_2 = \left[\begin{array}{c} u_{1,2} \\ u_{2,2} \end{array} \right]$$

and $\mathbf{S}$ is an [2,2] diagonal matrix of the type:

$$\mathbf{S} = \left[\begin{array}{cc} \mathbf{s}_1 & 0 \\ 0 & \mathbf{s}_2 \end{array} \right]$$

Scalars s_i are all real and positive and can be ordered as $s_1 > s_2$. Now we have all the elements to show how this decomposition correctly represents the MM that quantify the aggregate scale effects and the associated structures of the impact of a shock in disposable income on total output. In fact if we express the actual vector $\mathbf{f}$ in terms of the structures identified by matrix $\mathbf{V}$, we obtain final demand vector, $\mathbf{f}^0$, expressed in terms of the structures suggested by the $\mathbf{R}$:

$$\mathbf{f}^0 = \mathbf{V} \cdot \mathbf{f} \tag{17}$$

On the other hand we can also express total output according the output structures implied by matrix $\mathbf{R}$:

$$\mathbf{x}^0 = \mathbf{U}^T \cdot \mathbf{x} \tag{18}$$

Equation 16 then becomes through equations 17 and 18:

$$\mathbf{x}^0 = \mathbf{S} \cdot \mathbf{f}^0 \tag{19}$$

which implies:

$$x_i^0 = s_i \cdot f_i^0 \tag{20}$$

where $i = 1, 2$. We note that matrix $\mathbf{R}$ hides 2 fundamental combination of the outputs. Each of them is obtain multiplying the corresponding combination of final demand by a predetermined scalar which has in fact the role of aggregated Macro Multiplier.

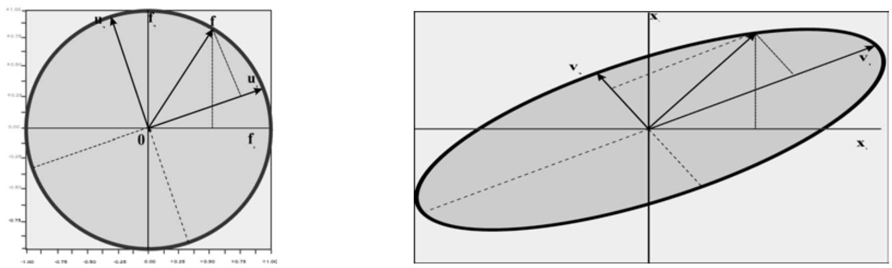
The complex effect on the output vector of final demand shocks can be reduced to a multiplication by a constant s_i.

The structures we have identified play a fundamental role in determining the potential behavior of the economic system, i.e. the behavior of the system under all possible shocks. We can in fact evaluate which will be the effect on output of all final demand possible structures. This is easily done imposing in equation 16 a vector whose modulus is constant, say equal to one, but whose structure can assume all possible configurations. If vector **f** in equation 16 is such that

$$\sqrt{\sum_j f_j^2} = 1 \tag{21}$$

then geometrically we mean that the final demand vector describes a sphere of unit radius (the unit circle).

Figure 4: Unit circle and corresponding ellipsoid for disposable income



a) changes final demand b) corresponding changes output industry

It rotates around the origin, as in Figure 4(a), assuming all the possible structures, including those implied by the columns of matrix **V**. Correspondingly the vector of total output will describe an ellipsoid with semi-axes of length $s_1, \ldots, s_m$, oriented according the directions designated by the columns of matrix **U**, as in figure 4(b). This ellipsoid is sometimes called the isocost of final demand control.

When final demand vector crosses a structure in **V**, the vector of total output crosses the corresponding structure in **U** and the ratio between the moduli of the two vectors is given by the corresponding scalar s. Singular values $\mathbf{s}_i$, then, determine the aggregated effect of a final demand shock on output. For this reason we will call them macro multipliers. These MM are aggregated, in the sense that each of them applies on all components of each macroeconomic variables taken into consideration, and are consistent with the multi-industry specification of the model[2].

In our original $[m,m]$ model, we can than say that, given our matrix **R**, we are able to isolate impacts of different (aggregate) magnitude,

[2]Given the problems connected with aggregation in multisectoral models, this feature of singular values s_i is not of minor relevance. They are aggregated multipliers consistently extracted from a multisectoral framework and their meaning holds both if we speak in aggregated or disaggregated terms.

since that MM present in matrix $\mathbf{R}$, $\mathbf{s}_i$ can be activated through a shock along the demand structure $\mathbf{v}_i$ and its impact can be observed along the output structure $\mathbf{u}_i$. In section 5 we will show the attempt to verify of MM approach.

5 Empirical analysis of Macro Multiplier Approach

The empirical analysis is performed thought a regional SAM what it has been built for the region Marche (Socci, 2004). The macro multiplier analysis starts from each cell of the table 1 where it shows the growth of the i^{th} industry output, $\mathbf{x}_i$, caused by a demand impulse of 1, $\mathbf{f}_i$, in the demand of goods produced by the i^{th} industry, as in classical SAM approach. The twelfth column shows the row sum which represent the total effect on the i^{th} industry output of a unitary final demand shock, $\mathbf{x}$, shown in column thirteenth, $\mathbf{f}$. The last row presents the column sums of the table and gives the effect on all the industry on outputs of a unit change in demand of goods produced by the i^{th} industry.

Table 1: Direct and indirect effects of final demand on total output

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	Sum	f
x_1	1.28	0.09	0.06	0.04	0.04	0.21	0.20	0.07	0.08	0.08	0.06	2.22	1
x_2	0.11	1.23	0.25	0.08	0.08	0.07	0.18	0.11	0.12	0.15	0.11	2.46	1
x_3	0.04	0.05	1.04	0.02	0.03	0.02	0.06	0.04	0.04	0.05	0.04	1.43	1
x_4	0.32	0.42	0.27	1.22	0.40	0.17	0.52	0.40	0.33	0.39	0.23	4.67	1
x_5	0.51	0.65	0.34	0.26	1.38	0.24	0.72	0.46	0.54	0.57	0.27	5.93	1
x_5	0.26	0.27	0.17	0.11	0.12	1.19	0.33	0.20	0.22	0.22	0.18	3.26	1
x_7	0.03	0.05	0.04	0.02	0.02	0.02	1.06	0.03	0.04	0.04	0.03	1.39	1
x_8	0.98	1.27	0.73	0.55	0.61	0.50	1.44	2.07	1.10	1.16	0.55	10.95	1
x_9	1.42	1.98	1.36	0.95	1.05	0.88	2.30	1.44	2.63	1.81	1.29	17.10	1
x_{10}	0.16	0.25	0.18	0.10	0.12	0.10	0.27	0.16	0.19	1.20	0.16	2.88	1
x_{11}	0.15	1.18	0.58	0.25	0.31	0.22	1.19	0.43	0.39	0.45	1.49	6.64	1
Sum	5.28	7.44	5.02	3.6	4.16	3.62	8.27	5.40	5.67	6.11	4.41	58.975	11

Applying Singular Value Decomposition on matrix shown in table 1 we obtain eleven singular values, table 2. As defined in section 4 singular values can be called MM and Table 2 shows the multipliers which are present in matrix $\mathbf{R}$. MM 1 (7,35) is the dominating one for its order of magnitude. This means that a final demand vector change produces a change on the output vector 7.35 times greater. MM from 2 to 6 amplify the effect of the shock, while the last four MM reduce it.

In the graphs in Figure 5(a) we have reported, with the black histogram, the input structure $\mathbf{v}_1$ able to activate MM s_1, and, with the white histogram, the corresponding effect on industrial outputs given by s_1 $\mathbf{u}_1$. While in Figure 5(b) and 5(c) the same is done for $\mathbf{v}_2$, s_2 $\mathbf{u}_2$ and $\mathbf{v}_3$, s_3 $\mathbf{u}_3$.

Table 2: Macro Multipliers relative to matrix **R**

s_1	7.35
s_2	1.57
s_3	1.21
s_4	1.13
s_5	1,09
s_6	1.05
s_7	1.00
s_8	0.99
s_9	0.93
s_{10}	0.84
s_{11}	0.69

The information emerging from Figures 5 can help in designing demand policies which are consistent with the observed structure of the interindustry interactions indicating the structure which are easiest to control. As an example let us refer to Table 3.

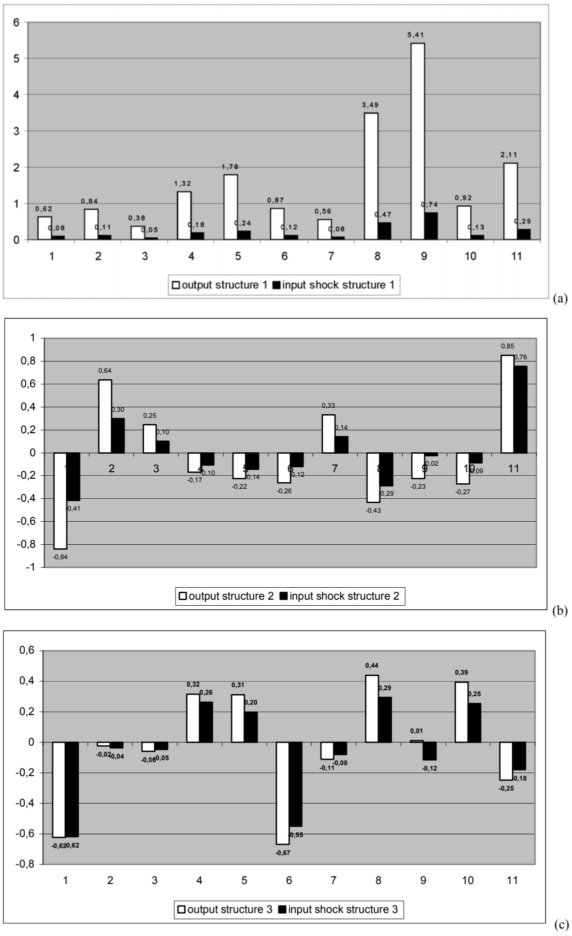
Table 3: Direct and indirect effects on total output of final demand according structure 1

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	x	f	x^2	f^2
x_1	1.11	0.12	0.05	0.02	0.03	0.12	0.29	0.07	0.10	0.09	0.05	2.00	0.86	4.20	0.74
x_2	0.10	1.58	0.20	0.05	0.06	0.04	0.25	0.12	0.15	0.16	0.09	2.70	1.28	7.70	1.65
x_3	0.04	0.07	0.83	0.01	0.02	0.01	0.09	0.04	0.05	0.05	0.03	1.20	0.80	1.50	0.64
x_4	0.28	0.54	0.22	0.75	0.29	0.10	0.75	0.43	0.43	0.43	0.19	4.30	0.61	19.20	0.37
x_5	0.44	0.84	0.27	0.16	1.00	0.13	1.03	0.50	0.69	0.62	0.22	5.90	0.72	34.80	0.52
x_6	0.22	0.34	0.14	0.07	0.09	0.65	0.47	0.22	0.28	0.24	0.14	2.80	0.54	8.20	0.30
x_7	0.03	0.07	0.03	0.01	0.02	0.01	1.52	0.04	0.05	0.05	0.03	1.80	1.43	3.40	2.06
x_8	0.85	1.63	0.58	0.34	0.44	0.27	2.07	2.25	1.41	1.27	0.45	11.00	1.08	133.0	1.18
x_9	1.22	2.55	1.09	0.58	0.76	0.48	3.30	1.56	3.37	1.98	1.06	17.00	1.28	322.0	1.63
x_{10}	0.14	0.32	0.14	0.06	0.09	0.05	0.39	0.18	0.24	1.32	0.13	3.00	1.09	9.30	1.20
x_{11}	0.13	1.52	0.47	0.15	0.22	0.12	1.71	0.47	0.50	0.49	1.21	7.00	0.81	49.0	0.66
Sum	4.56	9.58	4.03	2.20	3.01	1.98	11.88	5.88	7.26	6.70	3.60	60.67	10.55	593.7	11

Table 3 has been built on the basis of matrix **R** multiplying it by a vector of final demand which has the same modulus of **f** in Table 1 but of composition equal to $\mathbf{v}_1$.

Each cell shows the growth of the i^{th} industrial output, $\mathbf{x}_i$, caused by a demand impulse of f_j, as described in j^{th} row of the column **f** of the table. The last four columns show, respectively, the total output vector change, **x**, and the final demand shock, **f**. Here two columns, $\mathbf{x}^2$ and $\mathbf{f}^2$, have been added in order to calculate the squares of the industry values of vector **x** and vector **f**. This will facilitate the determination of the modulus of each vector, which will be done by taking the square root of the sum of each column. In fact when dealing with a vector representing a change it is convenient to refer to the commonly accepted measure of vector change which is the modulus of the vector. This is especially true if we want to take into consideration the possibility of considering also negative changes in some components of the vector.

Figure 5: Demand shocks structures and their impact on output composition



From the last two values we can appreciate that the final demand vector in Table 3 has the same module of that in Table 1 which are both equal to

$$\| f \| = \sqrt{\sum_i f_i^2} \tag{22}$$

hence $\sqrt{11}$ =3,317. While the output vector module is equal to

$$\| x \| = \sqrt{\sum_i x_i^2} \tag{23}$$

hence $\sqrt{593,7}$=24,36.
The ratio between the two modules is equal to $\| x \| / \| f \| =$

$24,36/3,317 = 7,347$ which is the value of multiplier s_1. This corresponds to a change in output of 60,674 which is higher than that obtained by chance in Table 1 that amounted to 58,975, and it is the highest performance the economy can attain. No higher performance can be attained.

Table 4 shows a similar application made with reference to the second multiplier s_2=1,5695. Here $\| x \| = \sqrt{27,10} = 5,2057, \| f \| =$

Table 4: Direct and indirect effects on total output of final demand according structure 2

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	x	f	x^2	f^2
x_1	-2.28	0.13	0.03	-0.01	-0.02	-0.12	0.14	-0.06	-0.04	-0.04	0.12	-2.15	-1.78	4.65	3.15
x_2	-0.20	1.66	0.13	-0.03	-0.04	-0.04	0.12	-0.10	-0.06	-0.08	0.19	1.56	1.35	2.43	1.81
x_3	-0.08	0.07	0.54	-0.01	-0.01	-0.01	0.04	-0.03	-0.02	-0.03	0.06	0.55	0.52	0.27	0.27
x_4	-0.57	0.56	0.14	-0.43	-0.19	-0.10	0.36	-0.36	-0.16	-0.22	0.42	-0.54	-0.35	0.29	0.13
x_5	-0.91	0.88	0.18	-0.09	-0.65	-0.13	0.50	-0.42	-0.26	-0.33	0.48	-0.75	-0.47	0.56	0.22
x_6	-0.46	0.36	0.09	-0.04	-0.06	-0.66	0.23	-0.18	-0.11	-0.12	0.32	-0.62	-0.55	0.39	0.31
x_7	-0.06	0.07	0.02	-0.01	-0.01	-0.01	0.74	-0.03	-0.02	-0.02	0.06	0.73	0.70	0.53	0.49
x_8	-1.74	1.71	0.38	-0.20	-0.29	-0.28	1.01	-1.89	-0.52	-0.66	0.99	-1.49	-0.91	2.23	0.83
x_9	-2.52	2.67	0.70	-0.34	-0.50	-0.48	1.61	-1.31	-1.25	-1.03	2.33	-0.12	-0.48	0.01	0.23
x_{10}	-0.28	0.33	0.09	-0.04	-0.06	-0.05	0.19	-0.15	-0.09	-0.69	0.28	-0.45	-0.57	0.20	0.33
x_{11}	-0.27	1.60	0.30	-0.09	-0.14	-0.12	0.83	-0.40	-0.19	-0.26	2.67	3.93	1.80	15.48	3.24
Sum	-9.37	10.02	2.60	-1.28	-1.97	-2.00	5.79	-4.93	-2.70	-3.49	7.92	0.60	-0.75	27.10	11

$\sqrt{11} = 3,317$ so that the module ratio is $\| x \| / \| f \| = 1,5695$. Here, as we could expect from the results shown in Figure 5(b) the shock slows down industries 1, 4, 5, 6, 8, 9 and 10 and expands industries 2, 3, 7 and 11. A shock with reverse sign would produce a reverse effect on the same industries. Revealing that the interactions in our economy create privileged sets of industries.

We have shown, through the use of numerical examples, that the parametric structure suggests the most effective demand policy, since whatever composition in the demand change other than v_1 causes less relevant results in terms of magnitude of the changes observed on industrial outputs. While the traditional Leontief multipliers analysis doesn't warrant that the results shown are the largest attainable.

6 Conclusions

Our attempt is to find out which is the better composition of the exogenous variable to obtain a particular effect on the objective variable. It allowed to identify particular indicators defined Macro Multipliers. This approach follows Leontief tradition and SAM multipliers, but it doesn't use the same hypothesis on the policy variable (unitary change), allowing to establish the line guides for policy makers.

The propagation analysis we propose is based on a decomposition that allows for the identification and quantitative determination of aggregated MM, which lead the economic interactions, and the structures of macroeconomic variables, that either hide or activate these forces. The analysis applied to an extended final demand-output loop that can be quantitatively tested forwarding a shock on a given macro-variable and observing the effects on another macro variable within the loop. It will identify the most efficient structure to the aim of the policy maker, given the framework of the economy. In particular, we use the multisectoral/multindustry model inspired from SAM scheme where we have a complete income circular flow extending a classical I-O model. The final inverse matrix shows structural economic characteristics.

Singular Value Decomposition method identifies allows a spectral analysis of our economy represented to inverse matrix of the our model. This method identifies which is the input structures associated to each output structures and they are ordered by singular values. Singular values give the magnitude of input on output structures and can be defined MM. From each of them, using input structures and applying it to final demand we could obtain output vector where the effect is quantified from MM. We could use MM also to generate the effect on particular industry, for example.

From policy stand point MM are relevant because they represent the potential effects of all the possible policies through industries (output generation) and sectors (final demand stimulus).

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A Social Accounting Matrix data

In this appendix we show the SAM data used above to perform the singular value decomposition. The SAM is presented by fundamental blocks and they are relative to different phases of income circular flow. All flows are expressed in billions of Italian 1996 liras.

In table 5 we observe intermediate consumption among industries.

Table 5: Intermediate consumption

	1	2	3	4	5	6	7	8	9	10	11
x_1 Agriculture	626.8	0	0	11	3	1214	155	222	169	4	66
x_2 Oil	61.2	955	321	272	43	60	16	261	402	92	198
x_3 Energy	34	11	99	122	34	35	7	159	171	36	86
x_4 Metal & chem. Products	227.7	17	46	3131	2001	120	103	4066	227	174	289
x_5 Machinery and Cars	40.6	1	39	115	1596	1438	4	946	253	7	704
x_6 Food	460.2	0	0	738	0	811	52	894	3379	12	168
x_7 Tobacco & Alc. Beverages	0.6	0	0	1	0	5	2	1	803	0	22
x_8 Manufacturing	37.6	12	70	600	590	142	77	7504	1771	130	665
x_9 Transport & Trade	433.7	48	83	1656	1608	649	185	3501	7018	681	1041
x_{10} Service market	0.1	1	13	29	629	1	2	8	14	0	0
x_{11} Service non market	0	0	0	0	0	0	0	0	0	0	0

In table 6 final consumption of Households Income Class to industries are shown.

Table 6: Final Consumption of Households Income Class

	I_Income class	II_Income class	III_Income class	IV_Income class	V_Income class
x_1 Agriculture	54	116	330	205	17
x_2 Oil	75	160	456	284	23
x_3 Energy	74	158	449	279	23
x_4 Metal & chem. Products	75	221	944	469	23
x_5 Machinery and Cars	31	247	1652	672	8
x_6 Food	262	561	1598	993	80
x_7 Tobacco & Alc. Beverages	51	109	309	192	16
x_8 Manufacturing	300	879	3757	1867	90
x_9 Transport & Trade	1209	2588	7370	4581	371
x_{10} Service market	255	546	1555	966	78
x_{11} Service non market	54	116	330	205	17

In table 7 value added generation by industries is splitted between two value added factors. The first is wages and salaries and the second is other incomes.

Table 7: Value Added (Va) by industry

	1	2	3	4	5	6	7	8	9	10	11
Va_1	626.8	0	0	11	3	1214	155	222	169	4	66
Va_2	61.2	955	321	272	43	60	16	261	402	92	98

The value added by factors is subject to primary income distribution and in table 8 each flow (wages and salaries, other incomes) is shown by institutional sectors (Households and Firms).

Table 8: Primary distribution of Income by institutional sectors

	Va_1	Va_2
I_Income class	133	529
II_ Income class	2006	875
III_Income class	10379	5303
IV_Income class	6998	6244
V_Income class	364	1451
Firms	0	12128

The income circular flow will show then transfers among institutional sectors from which derives the disposable income by institutional sectors. Table 9 presents the transfers from/to institutional sectors (Households and Firms).

Table 9: Secondary distribution of Income among institutional sectors

	I_Income class	II_Income class	III_Income class	IV_Income class	V_Income class	Firms
I_Income class	42	0	0	0	0	415
II_ Income class	0	4	0	0	0	1375
III_Income class	0	0	17	0	0	3988
IV_Income class	0	0	0	3	0	3309
V_Income class	0	0	0	0	0	251
Firms	41	160	553	380	12	3637

From income generation we have also indirect and import taxes which are represented in table 10.

Table 10: Government Indirect and Import Tax

	1	2	3	4	5	6	7	8	9	10	11
Indirect Tax	45	75	1	26	149	32	20	111	0	0	0
Import Tax	-309	1630	73	66	120	77	876	2048	1569	157	0

Secondary distribution of income shows also flows between government and other institutional sectors. In table 11 we have the government flows from other institutional sectors.

Finally, in table 12 there are government flows toward other institutional sectors: generally they are transfers without corresponding flows.

Table 11: Government flows from others institutional sectors

	I_Income class	II_Income class	III_Income class	IV_Income class	V_Income class	Firms
Government	203	1085	5344	4042	408	2585
Current Tax on Income	161	475	2205	1925	299	2242
Social transfers	40	604	3126	2108	110	0

Table 12: Government transfers

	Public Administration
I_Income class	1272
II_ Income class	2534
III_Income class	6653
IV_Income class	3400
V_Income class	606
Firms	5720