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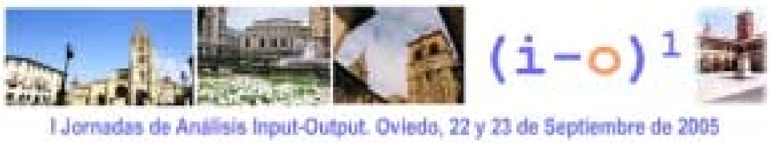

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There exists consensus about the relationship between linkage and development, and in respect of the fact that the process is led by very few production brands. However, there continues to be discussion regarding the definition and estimation of the so-called key sectors. Literature has mainly focused on reviewing the different conceptions or definitions of key sectors to determine what they are or should be. Authors have aimed to prove the advantage of some techniques as compared to others; nevertheless, in our opinion, none of them are competitors: they are complementary.

This paper studies key sectors in Spain [with a new approach] and logically different results are achieved. We discuss several methods used to estimate key sectors and we add some variations and suggestions, although we conclude that no single procedure is definitive.

Key Sectors: Big Coefficients and Important Coefficients in Spain

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Abstract

There exists consensus about the relationship between linkage and development, and in respect of the fact that the process is led by very few production brands. However, there continues to be discussion regarding the definition and estimation of the so-called key sectors. Literature has mainly focused on reviewing the different conceptions or definitions of key sectors to determine what they are or should be. Authors have aimed to prove the advantage of some techniques as compared to others; nevertheless, in our opinion, none of them are competitors: they are complementary.

This paper studies key sectors in Spain [with a new approach] and logically different results are achieved. We discuss several methods used to estimate key sectors and we add some variations and suggestions, although we conclude that no single procedure is definitive.

Key words: *Input-output, supply-side model, demand-side model, allocation coefficients, technical coefficient, key sector.*

Input-output analysis, linkages and models

The IO analysis has a long-standing tradition as much due to its existence for half a century as for having been the focus of constant debate. Just like other similar techniques, its development has taken place in stages that can be identified, more or less, according to their natural decades.

Those that may be called pioneer works appear at the end of the 1950s and are attributed to authors as well known as Chenery and Watanabe (1958), Rasmussen (1956) and Hirschman (1958). Given that, in matrix form, an input-output table can be expressed as a sum of rows $x=Ax + d$ or columns $x=xB + v$, with x being the total output, d the final demand and v the primary inputs, Chenery and Watanabe proposed the sum of the columns of the matrix A of technical coefficients (z_{ij} being the intermediate output of sector i to sector j) as a measurement of the *backward linkages* (BL), while also proposing the sum of the rows of matrix B of allocation coefficients $b_{ij} = \frac{z_{ij}}{x_i}$ as a measurement of the *forward linkages*, FL. These first multipliers were called *direct* since they only collected the relationships between production and distribution among the branches in the first place, without taking into account the following rounds of intermediary purchases that should have been produced to supply, in the most classic model by Leontief, an exogenous stimulus of final demand.

The sum of the columns of matrix A and of the rows of matrix B already allowed for the characterization of the branches according to their greater or lesser connection with the rest: *primaries* or *manufacturers*, in case they are, or not, very backward related, acquiring abundant inputs, and *intermediates* or *finals* in function of the location of their destinations.

Shortly afterwards, and to further the concept of the multiplier, Rasmussen[1] (1956) put forward the sums of the columns and rows of the Leontief inverse matrix L, $x = Ax + D = (\mathrm{I}-A)^{-1} D$, where $L = (\mathrm{I}-A)^{-1}$, which had the advantage of gathering together the direct and indirect effects. The new backward multipliers would show in this way the total output that **a country** would have to achieve in order to provide a unitary increase in the final demand of a branch *j*. For their part, the forward multipliers would reveal the output that **a branch** has achieved in the event that the final demand of all branches expanded into one unit. As a similarity of what was previously achieved with linkages of a direct nature by Chenery and Watanabe, distinctions between branches have been establishing themselves in a way such that those that have above-average effects would make up the most important group. These branches would be *key* or *strategic*. As the inverse *L* can also be approached as a sum of the identity matrix plus the successive rounds or powers of matrix *A* of coefficients, $L = \mathrm{I} + \mathrm{A} + \mathrm{A}^2 + \mathrm{A}^3 + \mathrm{A}^4$..., with progressively decreasing addends, it will be evaluated that the direct effects, brought together by matrix *A* -- like matrix *B* – will be much more important than the indirect effects, quantifiable by the sum $\mathrm{A}^2 + \mathrm{A}^3 + \mathrm{A}^4$..., with which the key branches obtained through the direct multipliers will not be very different from those obtained through the inverse2.

Hirschman (1958) also proposed using the inverse to calculate linkages expressly quoting Rasmussen and in his defence of “unbalanced growth as a strategy to overcome the drawbacks of a model which entailed, for Third World countries, lack of

[1]It is less known that Rasmussen also accompanied his multipliers with additional vectors that may clarify his results. In this way, on one hand, he was aware of the possibility of using weighted multipliers. On the other hand, he proposed using a coefficient of variation to better know whether or not the multiplier concentrated its effects on an excessively reduced number of branches. In the third place, he introduced the concept of *key sector*, and finally, the problems that the aggregation could have on the multipliers were guarded against.

[2] The power of the matrix is also taken as an indicator of the moment or round where an indirect relationship between the branches appears. Alternatively, this development can be carried out with matrix *B* of allocation coefficients, $G = \mathrm{I} + \mathrm{B} + \mathrm{B}^2 + \mathrm{B}^3 + \mathrm{B}^4$..., *G* being $= (\mathrm{I}-B)^{-1}$.

industrialisation and overdependence on the export of primary products, he suggested that “key industries could be useful to stimulate the development of these countries, in addition to reactivating economies in crisis3. Since Rasmussen’s indexes could measure what Hirschman called total backward linkages (TBL) and total forward linkages (TFL), some authors started using the term Rasmussen-Hirschman (R&H) multipliers for multipliers based on Leontief’s inverse.

In a second stage, able to be centered in the 1970s, a series of improvements on those that were then classic multipliers were proposed. It was accepted that the concept of *key sector* was wide and diverse, often depending on the objectives that were to be noted *ex ante*. The most appropriate measure of weight for each case were debated, since they determined, to a great extent, the results to be achieved [Bharadwaj (1966), Hazari (1970), Laumas (1975 and 1976), McGilvray (1977)]. On the other hand, several authors, although accepting the total backward multipliers, criticized the lack of reality on which the forward multipliers estimated with the Leontief inverse were based, in that they were sustained by a very unreal hypothesis—identical growth of final demand of each one of the branches, or that the different branches of the system were considered “having equal rights in the words of Rasmussen (p. 135).

[3] For Hirschman linkages will only represent a stimulus for development when the interdependence between sectors is causal. The interesting factor is precisely the causality that is implicit in the process of inducing supply or demand pressures. He pointed out the need to favour activities that had the biggest pull over others (thanks to their intersectoral connections), due to their capacity to produce imbalances between supply and demand that would act as market signals, revealing investment opportunities and stimulating, in consequence, the development of complementary activities. This way, the imbalance itself becomes a stimulus for investment and technical innovation as a result of bottlenecks and scarcity of resources that require new technical solutions, in what was a faithful reflection of the Hirschmanian idea of development as the result of imbalances.

The need to qualify and improve classic multipliers by adjusting Rasmussen's indexes with dispersion and weighting factors was generally accepted. Specifically, the most appropriate type of weighting in each case (the relative importance of VAB or of the final demand of branches, for example) and the need to find more sophisticated weights was a widely debated issue in the decade of the seventies, since the choice determined, to a great extent, the results to be obtained (Diamond (1974), Boucher (1976), Laumas (1975, 1976), McGilvray (1977) and Rao & Harmston (1979)). On the other hand, other authors, including Bharadwaj (1966), Jones (1976) and Hewings (1974), focused on aggregation problems, having observed that overly simplified tables tended to provide a smaller number of key sectors. Finally, Yotopoulos and Nugent (1973) wanted to assess Hisrchman's ideas by analysing the importance of key sectors in a series of developed and developing countries. The lack of evidence resulting from their work was challenged by Laumas (1976), Boucher (1976) Riedel (1976) and Jones[4] (1976), authors who generally criticised both the procedure and the inappropriate use of the sources.

Furthermore, the different effectiveness assigned to backward and forward linkages was also a matter of controversy. Even though several authors accepted total backward multipliers (possibly corrected), they criticised the lack of a real factual basis for forward multipliers, estimated with Leontief's inverse on the basis of a fully unreal hypothesis: identical growth of final demand of each branch, or that the system's

[4] Jones (1976), for example, focused his criticism on three points: Yotopoulos and Nugent did not account for all the indirect effects in their multipliers, they worked with total, not interior, coefficients, and lastly, they had used tables that were much too aggregrated. He also added that to assume that a key sector did not necessarily entail being dynamic or having high growth was a conceptual confusion.
At times the process of replies and counterreplies that served as the introduction of certain articles, such as this exchange between Yotopoulos and Nugent, revealed a certain amount of haste in some of the objections: "*a tempest in a teapot*, making use of the same expression used by the authors (1973, p. 337) to respond to the criticism they received.

different branches were considered to have*“equal rights*, as Rasmussen put it (1956, p. 129).

A thought-provoking alternative that raised expectations emerged when Augustinovics (1970), Jones (1976) and Beyers (1976) proposed substituting the FL for the sum of the rows of the inverse $G = (I-B)^{-1}$ of the distribution matrix B, in accordance with the supply model proposed by Ghosh (1958). Concretely, in an exchange of opinions in the *Quarterly Journal of Economics*, Jones (1976) criticized the methods in use. In his opinion, the sum of the row of the Leontief inverse did not provide a measurement of the forward linkages symmetrical with that provided by the sum of the column, being inclined towards the sum of the rows of the inverse of B. The interpretation of the multiplier would be the total output that a country must achieve in order to supply a unitary increase in primary inputs in a branch j. Augustinovics (1970) advocated the terms *input approach* and *output approach* to define the analysis of the IOT through their columns or rows (p. 251), indicating that both presented their own specific advantages. Also, contrary to common belief, the data from the study on several European countries showed that matrix B of allocation coefficients was not more unstable than matrix A of technical coefficients. The stability of the coefficients seemed to depend, rather, on the complexity or richness of the productive structure of the countries; “*the more complex forms seem to be more stable* (p. 261), he explained. In the following decade, several authors [Giarratani (1980), Bon (1986)] would also study the stability of the allocation coefficients, b_{ij}, coming to the conclusion that they were at least as stable as the technical coefficients, a_{ij}. In fact, the models are so similar that one can be expressed as a function of the other and the stability of both should therefore be very similar[5].

There is no need to insist that the backward and forward linkages mentioned above are simultaneous effects of a specific activity. In consequence, assessing them with demand and offer multipliers implies applying these multipliers simultaneously. In any case, the 1980s opened a third stage characterized by a group of critics [Giarratani (1980), Oosterhaven (1981)] of the forward multiplier now based on Ghosh´s inverse. Cella (1984) pointed out that the indexes obtained by the Leontief and Ghosh models would not be able to be combined due to the simultaneous inconsistency of the models based on the coefficients *A* or *B*. The supposition of a stable matrix *A* was not compatible with that of a stable *B* matrix, and vice versa (*Join Stability Problem*[6]). Given that matrix *B* could be expressed as $B = \hat{x}^{-1} A\hat{x}$, alternatively, $A = \hat{x} B \hat{x}^{-1}$, and in the case of the inverses $(I-A)^{-1} = \hat{x} (I-B)^{-1} \hat{x}^{-1}$, $(I-B)^{-1} = \hat{x}^{-1}(I-A)^{-1} \hat{x}$ - a change in the final demand would make the total output change and, as a consequence, *B* would change, *A* remaining fixed. It was not possible to suppose that the coefficients *A* and *B* were fixed at the same time, and therefore it was necessary to choose between them. Cella (1984), on the other hand, expressed his doubts about the adequacy of the *forward linkages*: "*In fact in the Rasmussen index for the sector j FL consists only of commodity j and is therefore completely different from the real nature of FL (which, if correctly evaluated, should be expressed in terms of all commodities)* (p. 76). That is to say, it should be considered as a multiplier of a productive branch, not of an economy.

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[5] During the following decade, several authors, including Giarratani (1980) and Bon (1986), also studied the stability of distribution coefficients b_{ij}, concluding that they were at least as stable as the technical coefficients a_{ij}.

[6] Dietzenbacher (1997) and Mesnard (1997) can be consulted on the topic.

Relationship between A and B

$Z = A\hat{x}$	$Z = \hat{x}\ B$
$A = Z\ \hat{x}^{-1}$	$B = \hat{x}^{-1} Z$
$A = \hat{x}\ B\ \hat{x}^{-1}$	$B = \hat{x}^{-1} A\ \hat{x}$

For his part, Oosterhaver (1988, 1989) stressed the criticism of the Ghosh model, repeatedly qualifying it as implausible. In his opinion, if the *B* matrix of allocation coefficients were admitted as fixed, then the "*input ratios vary arbitrarily and may, in principle, assume any value depending on the availability of supply. So the essential notion of production requirements, i.e., the production function, is actually abandoned* (1988, p. 207)[7].

Rose and Chen (1986, 1991) came out in defense of the model, proposing two classes of *joint stability*. The *absolute joint stability*, or "*the requirement that both production and allocation coefficients remain constant after an application of either the production or allocation version of the input-output model* (1991, p. 28), and the *relative joint stability* or "*the degree to which production coefficients of an input-output model approximate their original value after an application of either the production or allocation version of the model* (1991, p. 28). The absolute stability would not be possible more than as a trivial case but, for its part, the relative stability could be accepted as long as the coefficients changed in acceptable proportions. A case based on the economy of Taiwan showed the relative stability of the coefficients despite a serious impact on supply. The conclusion of this study, as well as that of other similar studies,

[7] Gruver (1989) proved to be in agreement with Oosterhaven, also rejecting the Ghosh supply model, and emphasizing that its acceptance would be just like recognizing that the production functions of the branches, placed in the *A* column, would not have any essential input, with all of them being perfectly substitutable, which would be a hardly probably supposition.

brought them to the defense of the idea that the degree of instability was low in the majority of cases, and therefore its use was more tolerable than the use of other widely-used models or procedures for analysis. Thus they added, to provide an example, the use of indirect estimation methods like RAS[8].

In the 1990s, a new era of renovation begun, and in this debate about the consistency of models the contribution of Dietzenbacher (1997, 2001, 2002) stood out, as he insisted that what the Ghosh model reveals are the variations in output as a consequence of the variation of the prices of the primary inputs, just as Leontief´s own model of prices does:

"For a long time, however, this solution was questioned since the supply-driven model was believed to be implausible. Recently, *Dietzenbacher (1997), showed that this model is interpreted as a price model. The multipliers then reflect the effect of an increase of one dollar in the primary costs (e.g. value-added, imports, labour-costs) of industry i on the total value of production. If forward linkages are viewed as measuring a sector's dependence upon other sectors as buyers of its output, these multipliers are of a forward nature. They measure how much the output value of all sectors together increases when the increased production costs – as induced by an initial increase in the primary costs of sector I – are passed on to the buyers of the output* (2002, p.126).

Therefore what turns out to be accepted today is that the Ghosh model is equivalent to Leontief´s price model. And, in the same way that Leontief´s quantity model has its "double in the Leontief´s prices model, the Ghosh prices model would have its "double quantity model, logically equivalent to the Leontief.

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[8] In the same way, Rose and Allison (1989) affirmed that the supply model could possibly be used as an approach as long as the changes in *A* were not excessively important. In fact, their estimations about *joint stability* based on the classic tables of Washington State showed that, through simulation, the alterations in *A* and *B* were of very little importance.

Even so, the problem of the simultaneous use of both multipliers has not yet been solved, given the lack of homogeneity mentioned earlier. The search for alternative procedures to determine key sectors continues. This paper analyses some of these alternatives in three different sections. The first very briefly refers to the *key sectors* of the Spanish economy as established with Rasmussen's classic criterion. These *key sectors* shall be used as reference to compare them with other methods. The second section analyses a method proposed by Dietzenbacher, based on eigen values. The last section analyses the use of important coefficients and proposes an alternative or variation of the traditional procedure. Nevertheless, first of all we have considerated convenient to do some considerations about the statistical source because of difficults we have founds.

Treatment of Statistical source

It has proven to be customary that comparison of input-output tables presents problems. Taking into account the double change that has occurred in recent times, the appearance of a new classification of activities (NACE 1993) and, above all, the application of the new European System of Accounts (ESA 1995), the difficulty increases. This paper is based on the last Spanish table, corresponding to 1995 (SIOT-95), which was prepared on the basis of these new methodologies and new classifications. We deliberately chose to work with domestic data since the main objective is to study a domestic productive structure, namely the one in Spain in 1995. Some of the difficulties we encountered are the following:

- In the first place, the new ESA establishes the achievement of several input-output tables: *supply tables, use tables* and finally, a *symmetrical* one that allows for realization of traditional studies based on its inverse. In the first, the supply of goods and services by product and type of supplier is shown, while in the second the use of goods and services by product and type of use is illustrated, or in other words, intermediate and final destinations. These two tables may reflect a productive structure with the desired and possible detail and do not have to be square. The *symmetrical table* is a square matrix that may show product by product or branch by branch of activity. However, the large part of statistical information that can be obtained from production units does not usually include an exhaustive detailing of input product by product. That is why in practice supply and use tables data is fallen back on, re-elaborating them until more analytical and synthetic information is made available, which is necessary for elaboration of the *symmetrical tables.* This job is accomplished using supplementary investigation and through the adoption of hypotheses about the structures of the inputs by product or by branch of activity. Elaboration of the symmetrical table therefore must solve and eliminate the question of secondary productions; in the previous system this was solved by redirecting the output through a vector of transfer to the branch of characteristic activity. It is now proposed, on the contrary, the complete transfer of outputs and inputs, thus resulting in a final structure that is very similar to those in the previous I-O tables, but more homogeneous and elaborate. But this new procedure also means that the new tables are in fact of a different nature, which is going to make it difficult to compare them with previous ones.

- On the other hand, the new ESA imposes a new valuation principle, basic prices, purer or closer at factor cost than the previous producers´ prices. This fact, if it can very well be considered an improvement in presenting data less contaminated by taxes and subsidies, makes notably difficult the comparison between tables, especially in the case of certain industrial branches, the ones affected by these excise duties.

- Finally, it is best to bear in mind that the conventions collected in the different ESA for the elaboration of the I-O tables may cause their values to change according to the degree of consolidation that has been achieved among the intraindustry transactions. In this way, it must be recalled that the previous ESA furthered total consolidation of the activities with a size of 3 NACE digits, with only few exceptions. On the contrary, in the current ESA it has been proposed as a general principle not to consolidate, since it is attempted to show all of the flows, although these may be intra-branch. This difficulty may be avoided by elaborating new domestic tables with zero diagonal elements in such a way to make them comparable. The intra-branch sales have also been subtracted from the total output so that the value added is not altered.

Key sectors in the 1995 SIOT in terms of Rasmussen's criteria

In accordance with Rasmussen's proposal, in Spain the results identified a relatively small number of key sectors, as compared to standard results in this kind of analysis, given that there are few activities that have above-average linkages which, furthermore, do not appear concentrated in a specific number of branches. In this special case, there may be about fifteen out of the sixty that exist, or perhaps a few more if less stringent requirements concerning concentration[9] – Table number 1 – are applied.

[9] Both a variation coefficient and a ratio of the first intermediate flows in relation to the total were used as indicators of the degree of concentration of branch purchases and destination. The FL was estimated over Gosh's inverse. If less stringent concentration requirements were used, *agriculture, fishing, beverages* and *construction materials* (branches 27 and 28) could also be included.

In addition, – and this is also standard – most key sectors turn out to be industries that provide intermediate outputs: *Non-Metallic Ores, Water Distribution Services, Wood and Cork, Pulp, Paper and Cardboard, Publishing Products, Chemical Products, Glass Products, Metallurgy, Metal Products*, and *Mechanical* and *Electrical Machinery*. There are also some intermediate services such as *Other Transport Related Services, Raw Material Recovery Services, Financial Intermediation Services, Machinery Renting Services* and *Market Public Sanitation Services*.

However, half of the branches that are key sectors under Rasmussen's conditions – or Chenery and Watanabe's[10] – are very small in terms of their importance in total production (they represent less than 0.8% of Spanish production) or added value. In consequence, interest arising from their strong connections is offset by their small values – *Non-Metallic Ores, Water Distribution Services, Wood and Cork, Glass Products, Raw Material Recovery Services, Financial Intermediation Services, Machinery Renting Services* and *Market Public Sanitation Services*.

Proper values

Proper values, as it is well known, are sets of scalars referred to with many different names: characteristic roots, latent roots, eigenvalues … Each one determines an associated vector, right and left, called eigenvector or autovector. The highest eigenvalue λ is dominant λ_D and its associated vector is known as the Perron vector. The right eigenvector is defined as a column vector that satisfies the equation: $B\ V_d = \lambda\ V_d$, where B is a square matrix – allocation coefficient one, for instance – V_d the eigenvector and λ the eigenvalue associated to V_d. The system could thus be expressed

[10] Bear in mind that there are very few differences between *key sectors* defined in terms of C&W's or Rasmussen's criteria.

as (B - λI) V_d = 0. Likewise, the left eigenvector is a row vector that satisfies V_i A = λ V_i, which when transposed gives A' Vi' = λ V_i'; A is a square square matrix – technical coefficient one, for example – and, as in the previous example, could be transposed as (A' - λ) V_i' = 0.

Dietzenbacher (1992)[11] has proposed using the elements of the right and left vectors associated with the dominant eigenvalue of matrices B and A, respectively, as appropriate indicators of intersectoral relations. In particular, the elements of eigenvector V_d associated with the dominant eigenvalue λ_D of matrix B could be used to measure forward linkages, whereas those of the left eigenvector V_i of matrix A could be used to estimate backward relations. Thus B V_d = λ_D V_i and V_i^t A = λ_D V_i^t . A and B, which are similar matrices, have the same autovectors and, consequently, the same determinant, given that the latter can be expressed as the product of the former. Likewise, the elements of the dominant vectors are the same in matrix A, in A^n and in L – or in B, in B^n and in G.

Eigenvectors can be calculated by first estimating the autovectors and then solving the previous systems for λ_D. But the elements of the vectors associated with λ_D can also be estimated in an approximate manner by raising matrix A – or B – to a sufficiently large power and adding up its columns – or B's rows. The results will be very small numbers that can be normalised by dividing them by the average of these addends. Below you can see that these normalised addends coincide with the elements of the dominant vectors that have also been normalised. *Key sectors* would thus be defined in the usual manner as the forward and backward linkages that are above

11 Dietzenbacher (1992, p. 420) "We propose another alternative method ... In the next section it is shown that the elements of the Perron vector, that is the eigenvector corresponding to the dominant eigenvalue of a matrix, can be used for measuring linkages. The right-hand Perron vector of the output matrix provides the forward linkage indicator, the left-hand Perron vector of the input matrix gives the backward linkage indicator.

average, thus being greater than one. This approximation reveals that, in fact, Dietzenbacher's proposal amounts to using a concept that is very similar to classic multipliers as indicators of branch linkage, although instead of adding up columns or rows of A and B, or L and G, columns or rows of A^n or B^n are added up.

To back his proposal Dietzenbacher works with a series of small (13x13) Dutch IO tables which, we believe, may condition the results too much. He compares traditional multipliers to eigenvalues and finds certain differences, establishing a ranking among branches which shows greater differences for forward linkages than for backward linkages. The reason for all these differences is that at times the concentration of intermediate purchases or sales of sectors leads to an overestimation of their linkages, given that some large multipliers may lose a major part of their virtuality in the event their linkage is concentrated on one or on just a few activities. On the contrary, a branch may show a seemingly low relation with other branches, although this fact may be compensated if it is related to a large number of them. In this case classic indicators will subestimate this activity's relations.

In fact, as you can see, these circumstances are well known in IO literature and constitute a traditional problem in linkage indicators. It has always been insisted that they should be accompanied by concentration measures to qualify the results. However, in Dietzenbacher's opinion, the novelty and capacity of eigenvalues is that they provide a ranking of branches that is more in keeping with reality, properly adjusting over and underestimations of classic indicators.

In our case, estimation of the eigenvalues of matrices derived from the 1995 SIOT and their comparison with classic multipliers has enabled us to address the issues we discuss below.

In general terms, there exists a similarity between the results of the alternative measures compared (eigenvectors, C&W and Rasmussen), which Dietzenbacher announced in the conclusions of his paper: "*Tables show that the results for the three methods are, in general, quite comparable* [Dietzenbacher (1992), p. 426]. Thus, most of the key sectors mentioned in the previous section (*Non-metallic Ores, Wood and Cork, Pulp, Paper and Cardboard, Publishing Products, Chemical Products, Glass and Glass Products, Metallurgy Products, Metal Products*, *Machinery and Mechanical Equipment, Other Transport Related Services, Financial Intermediation Services* and *Machinery and Equipment Renting Services*) maintain their status when the eigenvalue method is used. However, the branches *Water*, *Electrical Machinery* and *Market Public Sanitation Services* disappear from the list, whereas the branch *Secondary Raw Material Recovery Services* is added – table number 1.

However, we must add that there appears to be no confirmation, at least not unequivocally, of Dietzenbacher's (1992) statement to the effect that *eigenvectors* are a better indicator of interindustrial linkages in order to avoid problems of over or underestimation. One cannot conclude, on the basis of the data derived from the 1995 SIOT table, which is sufficiently large and complex (70×70), that this method acts correctly as a filter in the face of an excessive concentration at intermediate destinations or purchases of the different branches.

- Thus, beginning with *key sectors,* the linkages – in both directions – of the branches *Water* and *Market Public Sanitation Services* are **subestimated,** differing in this manner from Rasmussen's ranking. The branches *Wood, Paper, Transport related services* and *Renting of machinery* also reveal moderate BLeigen values and yet they do not have too many concentration problems in their purchases. Finally, the

FLeigen for the branch *Chemical* is below, though very near, average, despite the fact that it does not have a high concentration of output either.

On the other hand, some key sectors also have **overestimated** eigenvalues in connection with their concentration, although in this respect there are no cases where this occurs in both directions or only backwards. Thus, the forward relations are overestimated in three branches: *Non-metallic ores*, *Metallurgy* and in particular in *Raw materials recovery services*, where 82% of its intermediate output has a sole destination: *Metallurgy products*. Finally, under eigen criteria the remaining key sectors (5 of them) are properly valued according to the concentration of their purchases and output.

- As regards the BL and FL of the other branches that are not considered key, the application of Dietzenbacher's method also entails coincidences and differences, in particular as regards possible overestimations. It is thus important to highlight that although there are no cases in which a sector with a high eigenvalue (forwards or backwards) has no strong linkages, or does not have a high Rasmussen index, which amounts to the same, the results are not fully clarified with respect to the degree of concentration of the flows of these branches.

Cases of overestimation in the BLeigen include the branches *Meat Products* and *Dairy Products*, since in spite of having a high eigen, they have high but extremely concentrated-as expected- backward linkages in Agriculture. The same applies to *Insurance Services* and *Sanitation Services*. Cases of overestimation in the FLeigen are found in *Forestry Products, Coal, Crude Petroleum, Cement, Iron Ores* and *Financial Intermediation Services*, where despite having very high eigenvalues there is a considerable concentration in their destinations.

Finally, in the cases of the branches *Railway Transportation Services* and *Air Transport Services*, FLeigen are more appropriate than the values of classic multipliers, taking into account the ample destination or "relationships of these branches. On the other hand, although the activity *Electricity Production* sells to almost all other branches, it has a low FLeigen (0.58).

On the basis of what has been said above, it seems possible to formulate the following conclusions:

- As a general principle there exists substantial consistency between the classic methods and eigenvalues for classifying branches according to their linkages.
- Differences are more important on the side of overestimation than underestimation. In other words, the proposed method does not properly solve the traditional problem of concentration.
- Finally, forward overestimation is more frequent than backward overestimation.

For all these reasons, despite the great interest of eigenvalues they do not seem to be much better indicators than classic methods. Thus, the information regarding direct and indirect connections contained in an inverse matrix (Leontief's or Ghosh's), as the sum of the series of powers of a technical or allocation coefficient matrix, seems to be much greater than the information contained in an element of the respective series.

Sensitivity of coefficients

If we observe an intermediate matrix Z corresponding to any country we will notice that the number of large flows is small in comparison to the large number of others that are much smaller. Thus, for example, in the Spanish IO tables for the year 1995 – see Table number 2 – the 18 biggest intermediate flows comprise 25% of the total, whereas the first 81 entail half. Starting from here concentration is obviously lower and the first 300 have to be grouped in order to reach 75% of the total, or the first 500 to reach 85%.

Table 2

Distribution table of the values of Z

No. of coefficients	% of total intermediate consumption
18 biggest	25%
81 biggest	50%
294 biggest	75%
500 biggest	85%
700 biggest	90%
1050 biggest	95%

Table 3

Distribution table of coefficients A and B

	A	B
Nill	1,317	1,317
Under 0.1%	1,715	1,817
Between 0.1% and 0.5%	950	965
Between 0.5% and 1%	340	336
Bigger than 1%	578	465
Total	**4,900**	**4,900**
Average not nil	0.0073	0.0085

The sectors that have large intermediate flows are varied, although they do have some common non-excluding features – Table 1. On one hand, they are branches that the SIOT considerably aggregates – in accordance with existing official sartorial classifications – as in the case of *Agriculture*, *Construction Work* or *Other Business Services*. On the other, they have a natural connection with the other activities, as in the case of industries such as *Electricity Production*, *Metallurgy*, C*hemical* or *Machinery*, and services like *Trade*, *Telecommunication* or *Transports*.

Focusing now on coefficient matrices – Table 3 – it could be assumed that the biggest or more "important elements will be the ones that correspond to or are backed by the large intermediate flows, but as we shall see, this is not always the case. We must not forget that coefficients are ratios with regard to production and as a result they can be large simply because of the numerator or denominator. Furthermore, a coefficient a_{ij}

may be large but belong to a branch of little importance and in consequence its influence will be minor despite its large size. On the contrary, there can be coefficients that are smaller but that have far more influence if they are part of a particularly large branch.

Focusing on Spanish figures, if you compare the 500 most important flows of Z with the 500 biggest a_{ij} and b_{ij} coefficients you obtain considerable correspondence between them, since 339 large technical coefficients a_{ij} have their respective large z_{ij} – 68% – and, on the other hand, a very similar amount, namely 345 large b_{ij} distribution coefficients, coincide with a large z_{ij} – 69%.[12] It is thus clear that there exists a certain relationship between big coefficients and big intermediate flows, but that this is not a universal principle.

Sensitivity studies have tried to classify different coefficients in relation to their importance or influence, highlighting those than can cause the biggest change in production. In consequence, a difference has been established between "important coefficients and simply big coefficients. These important coefficients, often just a small number of them, have been called *Most Important Coefficients* (MICs) and depending on how many there are in each branch, the importance or influence of the latter within the productive system is assessed:

> *"The most important coefficients can be used to find out the industries having the most significant impacts on the output. This is done simply by calculating the number of coefficients which the industry has among the most important input-output coefficients. The rows and columns of industries must be considered separately. The row coefficients indicate mostly the importance of products of this industry for technological development of other industries. Product innovations and improvements as well as substitutions are then primarily concerned. The column coefficients indicate mostly the importance of the production method of this industry for demand of output of other industries. Technological development of the production process of the industry is then important*. Forsell (1988, p. 296).

The connections between coefficient matrices A and B, and their respective inverses L and G, have been analysed by means of developments made on the basis of

[12] On the other hand, of the 500 biggest a_{ij} and the 500 biggest b_{ij}, only 27 a and b coefficients do not coincide.

Sherman and Morrison's original work (1950). One of these developments is the technique of *tolerable limits,* introduced by Sekulic (1968) and Jilek (1971) and developed by Schintke & Stäglin (1985), Sebal (1974) and Aroche Reyes (1996), amongst others. The idea is to identify coefficients whose relative variations provoke a bigger error or deviation in terms of total production of the branches of activity. Thus, a coefficient a_{ij} is important if a variation of the coefficient under 100% provokes a change that is greater than a pre-established level p% – 0.5% or 1% is generally used – in the total production of some of the branches.

The formula that is normally used to measure the sensitivity of coefficients can be expressed as $w_{ij}\left(p\right) = a_{ij}\left[l_{ji}\,p + 100\left(\frac{l_{ii}}{x_i}\right)x_j\right]$, where w_{ij} is the degree of importance of coefficient a_{ij}, p the maximum percentage of variation that it will provoke in the production of any sector x_j (in other words, "*acceptable* limit of error), l_{ij} an element of the inverse matrix $(I-A)^{-1}$ and x_j the production of sector j. Obviously, the bigger the value of w_{ij} is, the more significant the intermediate purchases j makes from i, in other words, coefficient a_{ij}, will be.

An inverse of elasticities $r_{ij} = \frac{1}{w_{ij}}$ can also be defined as the percentage of change in a coefficient necessary to vary the production of any sector at a specific p. The production *sensitivity coefficient* would correspond to this quotient, which would be progressively smaller as the importance of a_{ij} increased:

> *if a percentage error in the cell of less than 100% – i.e. the degree of sensitivity is rij < 100% – induces a prespecified change of p per cent in at least one sectorial gross output ... All other intermediate cells or input coefficients being characterized by an acceptable error interval or a high degree of sensitivity rij (p) > = 100% will be classified as unimportant ... Empirical analysis proves it advisable to use a percentage for p which permits that all important cells include 90% of total domestic intermediate transactions (Schitke, J. & Stäglin, R. 1988, p. 48).*

And yet, going back to the definition of w_{ij}, in the case where self-consumption is eliminated from the matrix Z, as in our work, l_{ii}, element of the diagonal of the inverse, will be equal or very close to 1. In addition, the result of the product $a_{ij} \frac{x_j}{x_i}$ will be allocation coefficients b_{ij}. In consequence, in the formula stated above very small lij $\frac{p}{100}$ numbers are going to be added to the said coefficients. Thus, MICs could in fact be considered as elements that are very close to the biggest or most significant b_{ij}, as a result of which for this purpose size and importance would be directly related. *If b_{ij} is big* – or r_{ij} small – *the corresponding a_{ij} is important*.

Estimation of coefficient sensitivity in the 1995 SIOT produces almost 500 MICs – 485 –approximately 10 per 100 of the table, which is a small but usual number in this kind of work: Schintke and Stäglin (1988), Forsell (1988). In Table 1 you can also see that the branches where these coefficients are concentrated, which once again should be described as *key*, are in many instances different from the ones established by the previous methods, although on this occasion the procedure followed is quite different. This time the main novelty is to include intermediate services which were not always taken into account by previous criteria based on multipliers, since said services lack strong backward linkage. In this sense the presence of the activity *Business services*, of growing importance in developed economies is paradigmatic. In sixteen occasions these services can make the production of other branches grow by 1% if it modifies its requirements in regards to them in (relatively) small amounts. From the other perspective, on twenty-two occasions the branch can grow by 1% if other branches increase their requirements towards it by a specific amount.

Big intermediate flows and important coefficients

Locating important coefficients has been resolved to a large extent through the estimation of MICs, but now it is possible to introduce an additional variable in the analysis, namely growth of production and therefore of added values and rents. Thus one could consider a procedure that would study the capacity of each coefficient to influence or stimulate economic production by means of the two most conventional models in IO literature, the demand model and Gosh's model. The procedure would consist in obtaining the most important technical and allocation coefficients, in the sense referred to, by means of the following iterative process:

In matrix A each and every coefficient would be successively varied by the same percentage, and after each change the new total production would be estimated by means of the traditional demand model: $x = (I-A)^{-1} f$. The same operation would be performed in matrix B for allocation coefficients, this time estimating production by means of Ghosh's model: $x = v (I-B)^{-1}$. Finally, a reasonable group of coefficients – once again 10% of the total – that caused the biggest effect on production would be selected. Having performed the previous operation, a *key sector* would be defined as the productive activity which stands out due to the number of *impulse coefficients* it has or, in other terms, as an activity that can provoke significant variations in overall economic production, under both Leontief's model and Ghosh's[13].

With the calculations made in the Spanish 1995 IOT, the comparison of the most important technical and allocation coefficients shows the following results:

There are sixteen possible *key sectors* corresponding to a range of activities that are major suppliers of intermediate outputs throughout the productive cycle – Table

[13] The most important technical coefficient was $a_{1, 12}$, livestock purchases made by the branch *Meat Processing* from the branch *Primary.* The second coefficient was $a_{40,54}$, purchases of *Real Estate Services* from the branch *Construction*.

number 4. Overall, these *key sectors* present a greater potential for offer than for demand; that is, they entail 61% and 48%, respectively, of the most important row and column coefficients, of a total of 500 specified in each case.

In a more specific sphere, *Construction* and *Hotel and Restaurant Services* stand out in the section of most important technical coefficients with 6% and 5%, respectively, of the total analysed. Whereas with allocation coefficients, *Business Services* and *Electricity Production* stand out with 10% and 7% of the total.

Table 4

Branches with the largest number of production impulse coefficients

Agriculture	*Construction*
Electricity Production	*Services for the Trade and Repairs of Vehicles*
Pulp, Paper and Cardboard	*Wholesale Trade and Intermediaries*
Chemical Products	*Services of Retail Trade*
Rubber Products	*Hotel and Restaurant Services*
Metallurgy Products	*Other Transport-Related Services*
Metal Products	*Financial Intermediation Services*
Machinery and Mechanical Equipment	*Other Business Services*
Services of Other Land Transport	

In the industrial sphere *Electricity Production*, *Chemical Products* and the *Metallurgy Cluster* stand out, whereas the presence of seven tertiary activities reflects the size and increasing dynamism of general intermediate services which, in so far as they are necessary for most sectors, entail higher levels of interdependence. Once again, their presence here and absence from more classic criteria based on multipliers is due to their lack of big backward linkages.

Before concluding this section, we want to stress how revealing it is to compare these impulse coefficients with the intermediate flows, as it is quite likely that there existed a prior suspicion that the important coefficients would be the ones backed by a large z_{ij}. In fact, if before there was a coincidence of about 70% between the respective MICs – or b_{ij} – and the respective z_{ij}, now the result is that the correspondence reaches

95% in the case of backward impulse coefficients and 93% for those with forward impulse coefficients; only 56 coefficients – 22 and 34 respectively – will be able to increase production significantly without depending on an outstanding intermediate flow. Thus, almost all the *key sectors* obtained this way coincide with the branches that have the biggest intermediate flows – Table 1. And, in fact, the details provided by the different procedures used to analyse the economy's productive structure are to be found, as is logical, in the information contained in the matrix of intersectoral flows, *Z.*

MICS, a reconsideration

Following the previous digression, it is perhaps a good idea to go back to MICs. The authors mentioned above, who introduced or worked with MICs, classified them in terms of their size/importance, establishing different groups amongst which they highlighted, for example, those with the smallest $r_{ij,}$ (less than 20%) because of their special relevance, calling them "the most important of the important ones. But as with any other rate, interpretation of elasticity is a delicate issue and can reflect situations that differ greatly. For this reason it is useful to analyse it always in the context of the absolute numbers on which it is based or with which it is calculated.

To begin, a careful analysis of the 1995 SIOT coefficients with the greatest elasticity might be the proper manner in which to understand the sense of MICs better. Thus, for example, coefficient $a_{28, 40}$ which identifies the purchase of *Non-metallic Ore Products* by *Construction*, has a $r_{28, 40}$ with a big value, 1.14, whereas $r_{4,9}$ – corresponding to coefficient $a_{4,9}$ – *Coal* purchases by the production sector *Electricity* – is only a little bigger, 1.24. This means that it suffices to increase coefficient $a_{28,40}$ by 1.14% or $a_{4,9}$ by 1.24% to increase total production of *Non-metallic Ore Products* and of *Coal* by 1%, respectively. Both elasticities are amongst the most important to be found, and are also characterised by the relevance or importance of the flows they represent, flows that are very prominent in all economies.

However, the smallest r_{ij} that can be located is $r_{39,29}$ (1.18) related to the sale of scrap metal by the branch *Raw Material Recovery Services* to *Metallurgy*. Once again, an increase of coefficient $a_{39,\ 29}$ of just 1.18% causes recovery of scrap metal to grow by 1%. But the problem is that this activity, of great ecological importance, is one of the smallest in terms of production value to be found in the Table. Something very similar happens with coefficient $a_{5,8}$ which identifies the purchase of *Crude Petroleum* by the branch *Refined Petroleum* – $r_{5,8}$ = 1.38 – given that domestic oil production in Spain is insignificant, hardly noticeable.

Thus, if a branch is to be defined as key because it comprises a noteworthy group of small r_{ij} – row or column – it should also be highlighted that the branch will respond or cause to respond to productions of other branches with great elasticity or intensity, as a result of relatively small changes in its coefficients, although this will not necessarily entail significant increase of production, income or employment. Said increase will depend on other factors, primarily the absolute values of the flows that are behind the coefficients. One might think that nearly the same occurs with classic multipliers, such as Rasmussen's, for example. A high multiplier does not necessarily mean that production has to increase considerably; it does mean, however, that the branch has many connections with other branches. All of this is true, but it is a good idea to bring it up and have it in mind when analysing all these observations.

But to overcome this first problem raised – regarding the fact that elasticity may be seemingly interesting but that it is calculated with figures that are not important – it might be advisable to differentiate MICs depending on whether or not they are backed by large intermediate flows. For that purpose, a double division could be established between 70% of those backed by large flows, which would be accepted, and those that are not, which would be rejected. Finally, an obvious general principle could be established: *the smaller the* r_{ij} *of a MIC and the bigger the* z_{ij} *that backs it up are, the greater the importance of MIC will be.* But even this principle has its limitations.

Even though the definition of MICs is demanding and, as a result, only a small number are obtained, and even though their number is further limited by this new principle, it is still possible to find some MICs which are difficult to accept as such. The problem is now the contrary of the one dealt with earlier. If we accept that elasticity is tolerable when r_{ij} is less than 100%, even as a mere indicator, then we would have to accept the possibility of technical coefficients variations that cannot be accepted when the flow z_{ij} behind the coefficients is big enough or sufficiently important. If, as an example, we go over some of these coefficients, we would be admitting the possibility that an already significant supply of electric energy to certain branches could grow a lot more, or that the branch *Agriculture* could supply 80% more tobacco to the *Processed Tobacco Industry*, or that certain services such as *Other Transport Related Services* or *Other Business Services* could significantly increase their offer without encountering as a result technical difficulties or problems of capacity. This does not seem probable.

This is why we feel it is necessary to restrict even more the concept of MIC or, at least, be more demanding when it comes to choosing the most important ones. In our case, we have carried out a double selection of MICs and z_{ij} that back them in such a way that only the smallest r_{ij} , or those under 25%, have been accepted, together with the biggest z_{ij}, the ones comprising the top quartile. In this way the coefficients selected, that vary little or very little, could, however, increase (by 1%) production substantially. Examples of this type of coefficient selected could be the sale of livestock by the Farming sector to the Meat Processing branch – which in any case is the biggest intermediate flow of the whole 1995 SIOT – since a variation of 3.7% makes considerable primary production increase by 1%; or even the sale of beverages by the manufacturing industry to the branch *Hotels and Restaurants*, since an increase of this input by 1.5% leads to an increase of the industry's significant sales by the usual 1%.

To conclude, the result of this stringent process is that the number of coefficients has gone down to about fifty. These coefficients, which seemingly offer no doubts as to their validity, are insufficient to designate key branches, particularly if you take into account their logical dispersion throughout the Table.

Distance Matrix

The results obtained finally led to a synthetic qualitative analysis linking the study of interdependence with graph theory. As everyone knows, this rapprochement is possible by means of the process of transformation of coefficient matrixes into binary or adjacent matrixes; in other words, by replacing their values with ones and zeros in accordance with a predefined filter that makes it possible to indicate which intersectoral flows are assessed as important ones and which ones are not. Starting from this process, it is possible to obtain a series of matrices associated to a graph. In our case we have used the distance matrix, linked to the previously mentioned series of increasingly higher successive powers whose sum satisfies the total requirements, in other words, $L = I + A + A^2 + A^3 + \dots + A^{n}$.

The *distance matrix* informs regarding the order (moment) in which the (direct and/or indirect) relations appear within the process of powers: in other words, the criteria followed consists of replacing elements other than zero resulting from the said process by a number that is indicative of the round in which the relation becomes apparent. The boxes of the matrix obtained this way will contain a 1 if there is a direct $i \to j$ relation; 2 if there is only a second order indirect relation (in the second productive round); 3 if the relation appears in the third round, ...; *ad infinitum*, in the case of total independence[14].

$$E(D) \to \left\{ \begin{matrix} e_{ij} = 0 & principal\ diagonal \\ e_{ij} = \infty & \text{sectores sin } relación \\ e_{ij} = 1 & relación\ directa \\ e_{ij} = 2 & relación\ de\ segundo\ orden \\ \cdots\ \ \cdots & \cdots \quad\quad \cdots \end{matrix} \right\}$$

[principal diagonal / sectors without relations / direct relation / second order relation]

The columns of the final result will show the total (direct and indirect) relations of each activity and the moment they occurred. Thus the type of link existing between two sectors is labelled, with direct relations having necessarily a greater importance and reflecting the existence of a stronger economic interrelation. For this reason, the number of first order relations a productive branch has is a good indicator of its importance, with increasingly fewer economically significant links as the number of the power increases (those of second order relations will be more valuable than those of value 3, the latter more valuable than the ones that appear in the fourth round, and so on). In this way, as an indicator of the relative category of each branch, the harmonic mean of the

[14] On the other hand, it is obvious that these elements of the distance matrix e_{ij}, correspond to the elements of the so-called “order matrix *B*, that Yan and Ames (1965) designed in an intent to define an interrelation function for the productive system.

columns of the distance matrix based on A, or of the rows of the one based on B, could be an appropriate exponent. And, once again, if a branch has a low harmonic mean in both its row and its column, it could be defined as a key sector.

Table 5
Key sectors according to the different criteria used

		Rasmussen	Eigen	MICs	Impulse Coefficients	Distances	
21	Pulp, paper and cardboard	k	k	k	k	k	**5**
23	Chemical products	k	k	k	k	k	**5**
29	Metallurgy Products	k	k	k	k	k	**5**
30	Metal Products, except Machinery and Equipment	k	k	k	k	k	**5**
31	Machinery and Mechanical Equipment	k	k	k	k	k	**5**
49	Other transport related services	k	k	k	k	k	**5**
51	Financial intermediation services	k	k	k	k		**4**
9	Electricity production and distribution services			k	k	k	**3**
22	Publishing products, printed material	k	k			k	**3**
24	Rubber products; Plastic material products			k	k	k	**3**
42	Wholesale trade and intermediaries			k	k	k	**3**
46	Services of other land transport			k	k	k	**3**
58	Other business services			k	k	k	**3**
1	Agriculture …			k	k		**2**
7	Non-metallic non-energetic Ores	k	k				**2**
20	Wood and cork	k	k				**2**
26	Glass and glass products	k	k				**2**
40	General real estate construction work				k	k	**2**
41	Services for the trade and repairs of vehicles				k	k	**2**
43	Services of retail trade				k	k	**2**
44	Hotel and restaurant services				k	k	**2**
55	Services of renting of other machinery and equipmen	k	k				**2**
11	Collected and purified water	k					**1**
33	Electrical Machinery and Material	k					**1**
39	Secondary raw materials recovery services		k				**1**
50	Telecommunication services					k	**1**
61	Market public sanitation services	k					**1**
		15	**13**	**13**	**17**	**17**	

Having prepared the distance matrices for the Spanish economy, the results of the harmonic means appear in Tables 1 and 5. It should be noted that in the matrix of intermediate flows Z very small values close to zero have been eliminated, since their conversion to value one in the distance matrix could distort the results. It is clear that the branches considered key by the *distance matrix* are quite similar, as was, once again, to be expected, to the ones obtained by means of the impulse coefficients. Thus this process could be accepted if, in turn, this meaning or version of key sectors is accepted.

Conclusions

In this paper we analyse and apply some of the techniques used to identify an economy's so-called key sectors. The theoretical debate regarding this issue in input-output literature is long-standing, even classical, we should say, as it goes back to its origins. Thus, in the course of an initial stage classic multipliers were developed (Chenery and Watanabe, Rasmussen), followed by a discussion of the different weightings to be applied, of the different concentration indexes to be used, of their applications to developing economies, of the relationship between key sectors and economic growth, etc.

A second stage began when Augusztinovics retrieved the so-called supply-driven model. By using this model it seemed possible, on the basis of Ghosh's inverse, to calculate forward multipliers that were more similar to the ones estimated with Leontief's inverse, which in turn provided a solution to a great deal of criticism, such as Cella's for example. However, many authors claimed that the model was not plausible (Oosterhaven) since it required accepting the possibility that technical coefficients could vary at random, thus altering the production functions that supported the tables.

In a more recent stage Dietzenbacher insisted that the so-called supply-driven model is merely an alternative manner of presenting Leontief's well-known price model and that, in consequence, instead of accepting increases of the amount the offer, alterations of the primary input prices should be accepted. This point of view has become fully accepted as the current paradigm, putting an end, for the moment, to a long on-going debate.

Dietzenbacher also proposed that the eigenvectors associated to the technical and allocation coefficient matrices were a good indicator of intersectoral relations. However, our estimates on the Spanish input-output table corresponding to 1995 did not provide evidence of all his proposals and conclusions. In this sense, even if there is a significant similarity between the key sectors that can be identified by means of the proposed method and those identified with classic multipliers, the traditional problems of over or undervaluation of linkages due to concentration or dispersion, respectively, of linkages between different branches do not seem to be solved any better.

An alternative way of identifying key sectors is through the so-called important coefficients or MICs, since a branch that has a considerable number of them will play, no doubt, a predominant role in an economy. About 500 MICs were obtained in the 1995 SIOT (10% of the table's elements), a number which is usual in that kind of study. But a more detailed analysis of these 500 components has revealed a double disadvantage. On one hand, there are occasions in which the importance of the coefficient is very high as regards its elasticity or capacity to increase production in a specific branch. However, the affected production may have no importance at all with respect to its absolute figures. On the other hand, if the affected input, the one that is behind the MIC, is very large, the probabilities that it will grow significantly – in any case less than 100%, as is standard in this kind of analysis – are minimum.

In our case, we have eliminated the MICs that we considered irrelevant or improbable, in the sense mentioned, with the disadvantage that their number has greatly dwindled, to just about fifty. This small amount has raised doubts regarding the possibility of using MICs to identify key sectors.

We have chosen to reintroduce in the study of key sectors the relevance of the absolute numbers that appear in the table. In other words, even though in the past the definition of a key sector stressed its degree of relations – linkages – with the other branches, it seems appropriate to qualify all of the results with the affected productions. Thus, using Rasmussen's own indexes, it is possible to find key sectors that can not be criticised in any way, except for the fact that their production is irrelevant in quantitative terms. Nor do they seem to have a major technological role, or any other of special significance.

In our case, we have varied each technical and distribution coefficient was varied in order to consistently recalculate, by means of Leontief's and Ghosh's models, total Spanish production. Once that was done, the trial's 500 most successful technical coefficients and its 500 most successful allocation coefficients were selected. In 90% of the cases, they had or corresponded to an equally large or significant intermediate flow. The results, which are logical, reveal how advisable it is to always take into account the intermediate flow's absolute matrix, which must support the results obtained. Finally, the concentration of these "impulse coefficients in certain branches provide an alternative manner of establishing key sectors, considered to be the ones with the greatest possibility of fostering economic growth.

Lastly, two distance matrices were established, on the basis of the successive powers of the 1995 SIOT technical and allocation coefficient matrices, by previously filtering or eliminating only the smallest or irrelevant intermediate flows. Likewise, the harmonic means of the rows and columns of these matrices made it possible to establish key sectors with results similar to the ones obtained with the previous method. The fact is that even though the definition and the procedures for the estimation of key sectors are open, it is clear that certain branches appear obstinately time after time, regardless of the method used, thus highlighting their singular importance.

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